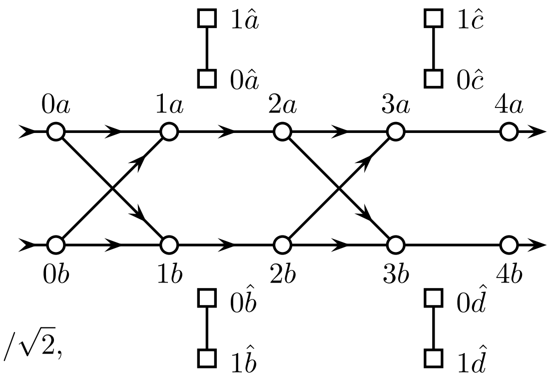
Toy Mach-Zehnder with Measurements



$S|0a\rangle = (1/\sqrt{2})|1a\rangle + (1/\sqrt{2})|1b\rangle,$

$S|0b\rangle = -(1/\sqrt{2})|1a\rangle + (1/\sqrt{2})|1b\rangle,$

$S|1a\rangle = e^{i\phi_a}|2a\rangle, \quad S|1b\rangle = e^{i\phi_b}|2b\rangle,$

$S|2a\rangle = (1/\sqrt{2})|3a\rangle + (1/\sqrt{2})|3b\rangle,$

$S|2b\rangle = -(1/\sqrt{2})|3a\rangle + (1/\sqrt{2})|3b\rangle,$

$S|3a\rangle = |4a\rangle, \quad S|3b\rangle = |4b\rangle.$

$|\psi_0\rangle = |0a\rangle, \quad |\psi_t\rangle = S^t|\psi_0\rangle.$

$|\psi_1\rangle = (|1a\rangle + |1b\rangle)/\sqrt{2}, \quad |\psi_2\rangle = (e^{i\phi_a}|2a\rangle + e^{i\phi_b}|2b\rangle)/\sqrt{2},$

$|\psi_3\rangle = [(e^{i\phi_a} - e^{i\phi_b})|3a\rangle + (e^{i\phi_a} + e^{i\phi_b})|3b\rangle]/2,$

$|\psi_4\rangle = [(e^{i\phi_a} - e^{i\phi_b})|4a\rangle + (e^{i\phi_a} + e^{i\phi_b})|4b\rangle]/2.$

$\odot$ Case 1. No detector. $\mathcal{H} = \mathcal{H}_p$.

$$\Pr([3a], t = 3) = (1/4)|e^{i\phi_a} - e^{i\phi_b}|^2$$

shows interference (probability depends on relative phases).

$\odot$ Case 2. One detector $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}}$, $T = SR$, $R = I$ except

$$R|1a, p\hat{a}\rangle = |1a, (1-p)\hat{a}\rangle,$$

where $p = 0$ (ready) or 1 (particle detected). Initial state $|\Psi_0\rangle = |\psi_0\rangle \otimes |0\hat{a}\rangle = |0a, 0\hat{a}\rangle$.

$\odot$ Case 3. Detector does not distinguish $1a$ and $1b$. Initial state $|\Psi_0\rangle = |0a, 0\hat{a}\rangle$.

$$R|1a, p\hat{a}\rangle = |1a, (1-p)\hat{a}\rangle, \quad R|1b, p\hat{a}\rangle = |1b, (1-p)\hat{a}\rangle.$$

$\odot$ Case 4. Two detectors $\hat{a}$ and $\hat{b}$; $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}} \otimes \mathcal{H}_{\hat{b}}$. Initial state $|\Psi_0\rangle = |0a, 0\hat{a}, 0\hat{b}\rangle$.

$$R|1a, p\hat{a}, q\hat{b}\rangle = |1a, (1-p)\hat{a}, q\hat{b}\rangle, \quad R|1b, p\hat{a}, q\hat{b}\,\rangle = |1b, p\hat{a}, (1-q)\hat{b}\,\rangle.$$

$\odot$ Case 5. Two detectors $\hat{c}$ and $\hat{d}$; $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{c}} \otimes \mathcal{H}_{\hat{d}}$. Initial state $|\Psi_0\rangle = |0a, 0\hat{c}, 0\hat{d}\rangle$.

$$R|3a, p\hat{c}, q\hat{d}\rangle = |3a, (1-p)\hat{c}, q\hat{d}\rangle, \quad R|3b, p\hat{c}, q\hat{d}\rangle = |3b, p\hat{c}, (1-q)\hat{d}\rangle.$$

- Consider a sample space of histories of the form

$$[\Psi_0] \odot \{[1a], [1b]\} \odot I \odot I \odot ??.$$

Is it consistent to have any of the detector states $[\hat{0}c], [\hat{1}c], [\hat{0}d], [\hat{1}d]$ among the possible events at $t = 4$? If not, what else?

$\odot$ Case 6. Weak detectors. Hilbert space $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_{\hat{a}} \otimes \mathcal{H}_{\hat{b}}$. Initial state $|\Psi_0\rangle = |0a, 0\hat{a}, 0\hat{b}\rangle$.

$$R|1a, 0\hat{a}, q\hat{b}\rangle = \alpha|1a, 0\hat{a}, q\hat{b}\rangle + \beta|1a, 1\hat{a}, q\hat{b}\rangle, \quad R|1b, p\hat{a}, 0\hat{b}\rangle = \alpha|1b, p\hat{a}, 0\hat{b}\rangle + \beta|1b, p\hat{a}, 1\hat{b}\rangle,$$

where $|\alpha|^2 + |\beta|^2 = 1$.

- Interference will be seen in those cases in which neither the $\hat{a}$ nor the $\hat{b}$ detector has triggered.
- The following family augmented with histories of zero weight is consistent:

$$\begin{array}{ccccccc} [\Psi_0] & \odot & \{[1a], [1b]\} & \odot & I & \odot & \{[1\hat{a}], [1\hat{b}]\} \\ [\Psi_0] & \odot & \{[\chi^1], [\chi^2]\} & \odot & I & \odot & \{[3a], [3b]\} \otimes [0\hat{a}] \otimes [0\hat{b}] \end{array}$$

where $|\chi^1\rangle = (|1a\rangle + |1b\rangle)/\sqrt{2}$ and $|\chi^2\rangle = (|1a\rangle - |1b\rangle)/\sqrt{2}$.