

33-445 Advanced Quantum Physics Fall Semester, 2012
Assignment No. 11. Due Friday, November 16

COURSE WEB PAGE:

<http://quantum.phys.cmu.edu/quad/>

READING: Sources

Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed

READING: Topics

Schrödinger equation in one dimension: Townsend, Sec. 6.8

Particle in a box: Townsend, Secs. 6.8, 6.9

Probability current: Townsend, Sec. 6.10

Gaussian wave packet: Townsend, Sec. 6.6

Scattering: Townsend, Sec. 6.10

Tunneling: Townsend, Sec. 6.10

Summary for particle in one dimension: Townsend, Sec. 6.11

Note: The material at the beginning of Townsend's Sec. 6.1 (Heisenberg microscope) and the entire Sec. 6.7 is a bit old-fashioned: not exactly wrong, but also not exactly right. Footnote 12 on p. 211 can be read as an admission that there is something fishy about Townsend's armwaving use of Heisenberg uncertainty. However, Example 6.5 gives a straightforward (and correct) calculation of an interference pattern.

READING AHEAD:

Harmonic oscillator using raising and lowering operators: Townsend, Secs. 7.1 to 7.4.

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

2. Consider a particle confined to a one-dimensional box (square well with infinitely high walls) $-a/2 \leq x \leq a/2$. The ground and first excited state wave functions are given by

$$\epsilon_1(x) = \sqrt{2/a} \cos(\pi x/a), \quad \epsilon_2(x) = \sqrt{2/a} \sin(2\pi x/a).$$

Assume that at $t = 0$ the wave function is

$$\psi(x, 0) = (1/\sqrt{2})(\epsilon_1(x) + \epsilon_2(x)).$$

a) Calculate $\psi(x, t)$ using a dimensionless time variable $\theta = 2\pi t/\tau$, where τ is the period of the motion, the time in which $\psi(x, t)$ returns to its original value apart from an overall phase. You should relate τ to the energies of the first and excited state.

b) Compute the probability density $\rho(x, \theta)$. Check that it is periodic: $\rho(x, \theta + 2\pi) = \rho(x, \theta)$. Sketch $\rho(x, \theta)$ (or make a computer plot) as a function of x for $\theta = 0, \pi/2, \pi$ and $3\pi/2$.

c) Find an expression for the probability current $j(x, t)$ and check that

$$-\partial j/\partial x = \partial \rho/\partial t.$$

Sketch $j(x, t)$ for some time at which it is not zero.

3. a) Show that for a particle in one dimension the average value of the momentum $\langle P \rangle_t$ at some time t is related to an integral of the probability current $j(x, t)$ at time t .

b) Evaluate the probability current $j(x)$ and the local velocity

$$v(x) = j(x)/|\psi(x)|^2$$

in the case in which

$$\psi(x) = C \exp[ik_0 x - x^2/2a^2],$$

where k_0 is real, and C is chosen to give the correct normalization. Do your answers seem to make sense? Discuss.

4 a) Find the local velocity $v(x, t) = j(x, t)/|\psi(x, t)|^2$ for the (initially) Gaussian wave packet $\psi(x, t)$ as it develops in time, using the formula on p. 209 in Townsend. Express your answer using the dimensionless time variable $\theta = \hbar t/ma^2$. (You may also find it convenient to replace x with the dimensionless $\xi = x/a$.) [Hint 1: When θ is real, $(1+i\theta)^{-1} = (1-i\theta)/(1+\theta^2)$. Hint 2: The fact that you divide $j(x, t)$ by $|\psi(x, t)|^2$ in the process of computing $v(x, t)$ will allow you to throw away some messy things without having to actually calculate them.]

b) For large $\theta \gg 1$ find a simple approximate expression for $v(x, t)$. Look for an arm-waving explanation in terms of the motion of a classical particle which starts at $x = 0$ at time $t = 0$, but with a random velocity (i.e., an initial “quantum” uncertainty).

5. Townsend Problem 6.19.