33-445 Advanced Quantum Physics Fall Semester, 2012
Assignment No. 10. Due Friday, November 9

COURSE WEB PAGE:
http://quantum.phys.cmu.edu/quad/

READING: Sources
Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed
CQT = Consistent Quantum Theory. Individual chapters at: http://quantum.phys.cmu.edu/CQT/

READING: Topics
Measurements: CQT Secs. 17.1, 17.2, 17.5, 18.1, 18.2.
Wave functions of position and momentum: Townsend, Secs. 6.1 to 6.6;   CQT Secs. 2.1 to 2.4.
Note: The material at the beginning of Townsend's Sec. 6.1 (Heisenberg microscope) and the entire Sec. 6.7 is a bit old-fashioned: not exactly wrong, but also not exactly right. Footnote 12 on p. 211 can be read as an admission that there is something fishy about Townsend's armwaving use of Heisenberg uncertainty. However, Example 6.5 gives a straightforward (and correct) calculation of an interference pattern.

READING AHEAD:
Bound states and scattering in one dimension: Townsend Secs. 6.8 to 6.11

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

2. Two spin-half particles a and b are prepared in an entangled state

$$|\psi_0\rangle = \big(|z_a^+, z_b^+\rangle + |z_a^+, z_b^-\rangle + |z_a^-, z_b^+\rangle\big)/\sqrt{3}.$$

Particle a is sent to Alice and b to Bob, who are a long distance apart. Both are contemplating carrying out spin measurements on the corresponding particles. Assume both Alice and Bob are competent experimentalists, and that they both know $|\psi_0\rangle$, but of course they do not know the results of their measurements before carrying them out.

a) Use an appropriate density operator to calculate the probabilities that if Bob measures S_{bz} he will get $+1/2$ or $-1/2$, and the same for measurements of S_{bx} and S_{by}. Note that since $|\psi_0\rangle$ is symmetrical when a and b are interchanged, these probabilities are also valid for the corresponding measurements when carried out by Alice on particle a.

b) Alice carries out an S_x measurement on particle a. Write out the expansion of $|\psi_0\rangle$ in terms of the basis $|x_a^+\rangle$ and $|x_a^-\rangle$. Use this expansion to answer the following question: Suppose her measurement outcome indicates $S_{ax} = +1/2$. Based on this (additional) information, what probabilities should she assign to the outcome of an S_{bz} measurement for particle b? Same question if her measurement outcome is $S_{ax} = -1/2$.

c) Show that the results in (b) are consistent with those in (a) in the sense that the overall probability that S_{bx} has a particular value should not depend upon Alice's measurement. That is, check that $\Pr(x_b^+) = \Pr(x_b^+ \mid x_a^+)\Pr(x_a^+) + \Pr(x_b^+ \mid x_a^-)\Pr(x_a^-)$, and the same with x_b^- in place of x_b^+.

d) After carrying out the S_x measurement on particle a, Alice phones Bob, who has not yet carried out a measurement on his particle, and tells him that she has carried out an S_{ax} measurement, but does not tell him what she found. Should he change the probabilities he would assign to an S_x measurement on particle b? What is Alice also tells him the measurement outcome?

3. Define the wave function $\psi(x)$ as

$$\psi(x) = \begin{cases} c(a-x) & \text{for } 0 \le x \le a, \\ c(a+x) & \text{for } -a \le x \le 0, \\ 0 & \text{elsewhere,} \end{cases}$$

where c is a positive constant.

a) What is the value of c so that $\psi(x)$ is normalized? Sketch $\psi(x)$ and the corresponding probability density $\rho(x)$.

b) Calculate the following probabilities:

$$\Pr(x > 0), \quad \Pr(-b < x < b) \text{ for } 0 < b < a, \quad \Pr(-a/2 < x < -a/4).$$

4. a) Let $|\phi\rangle$ and $|\psi\rangle$ be related by

$$\phi(x) = e^{ikx}\psi(x),$$

where k is a constant. Show that there is a simple relationship between the momentum probability distributions $\hat{\rho}(p)$ generated by $|\psi\rangle$ and $\hat{\sigma}(p)$ generated by $|\phi\rangle$. Use this to deduce a relationship between the averages $\langle p\rangle_\phi$ and $\langle p\rangle_\psi$.

b) Now compare position probability distributions in the case where, with x_0 a constant,

$$\hat{\phi}(p) = e^{ipx_0/\hbar}\hat{\psi}(p),$$

How is $\langle x\rangle_\phi$ related to $\langle x\rangle_\psi$?

5. Find two nonzero orthogonal kets $|\phi\rangle$ and $|\psi\rangle$, $\langle\phi|\psi\rangle = 0$ (make them simple) such that

$$|\phi(x)|^2 = |\psi(x)|^2.$$

(This shows that there is more to a quantum state than the position probability distribution.)

6. Find $F(x,p)$ such that for any $|\phi\rangle$ and $|\psi\rangle$ it is the case that

$$\langle\phi|\psi\rangle = \int \hat{\phi}^*(p)\psi(x)F(x,p)\,dx\,dp.$$