33-445 Advanced Quantum Physics Fall Semester, 2012
Assignment No. 7. Due Friday, October 12

COURSE WEB PAGE:
http://quantum.phys.cmu.edu/quad/

READING: Sources
Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed
CQT = Consistent Quantum Theory. Individual chapters at: http://quantum.phys.cmu.edu/CQT/

The following items are on the course web page. They were prepared for a previous course, so there will be some overlap with other notes, and they contain some things we will not be covering.

UTDBR = "Unitary Time Development and Born Rule"
STP = "Stochastic Processes"
PRQHI = "Probabilities for Quantum Histories I"
PRQHII = "Probabilities for Quantum Histories II" Secs. 1,2.

READING: Topics
Stochastic histories: CQT Ch. 8; STP
Born rule: CQT Ch. 9; UTDBR; PRQHI
Toy models: CQT Secs. 2.5, 6.3, 7.4, 12.1, 12.2; UTDBR
Consistency conditions and probabilities of histories: CQT Chs. 10 and 11; in particular Sec. 11.6. We will not be considering the most general case in Ch.11 (summarized in Sec. 3 of PRQHII), and will instead focus on chain kets, Sec. 11.6; PRQHII Secs 1, 2.

READING AHEAD:
Interference: CQT Ch. 13. Feynman *Lectures on Physics*, Vol. III, Ch. 1; also in Vol. I. Ch. 37

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

2. a) Find a product sample space of histories (that is, choose a particular decomposition of the identity at each time, and make up the histories using one of the projectors from each of these decompositions) at three times for a spin-half particle, which contains as one of its elements the history

$$[z^+] \odot [z^-] \odot [x^+]. \tag{A}$$

[Hint. There are 8 elements in the sample space.]

b) Find a projector P in the event algebra that is a product projector (tensor product of one projector at each time) which at one of the times tells you nothing (is the identity operator). Find another projector Q in the event algebra that cannot be written as a tensor product of three projectors. Find the negations $\tilde{P}$, $\tilde{Q}$, and also the conjunction $P \wedge Q$ and disjunction $P \vee Q$ of these projectors.

c) Construct a smaller sample space with only 5 elements containing the history (A), which is an appropriate decomposition of the history identity $\breve{I}$ when one is only interested in a particular initial event, in this case $[z^+]$. How is the event algebra in this case related to that of the sample space in (a)?

3. a) Consider a spin-half particle in zero magnetic field, so that $T(t,t') = I$, and let the initial state at t_0 be $|\psi_0\rangle = |z^+\rangle$. Consider a family of histories

$$[z^+] \odot \{[w^+],[w^-]\} \odot \{[x^+],[x^-]\}$$

at times $t_0 < t_1 < t_2$. For (i) $w = x$, (ii) $w = y$ and (iii) $w = z$, determine whether the family is consistent, and if it is consistent, find the four conditional probabilities

$$\Pr\left([w^\pm]_1, [x^\pm]_2 \,\middle|\, [z^+]_0 \right),$$

where the subscripts indicate the times of the events.

b) Next suppose that the time development operator is

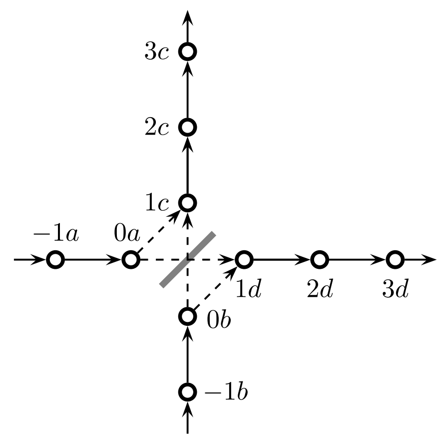
$$T(t_2,t_1) = T(t_1,t_0) = U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \begin{pmatrix} \langle z^+|U|z^+\rangle & \langle z^+|U|z^-\rangle \\ \langle z^-|U|z^+\rangle & \langle z^-|U|z^-\rangle \end{pmatrix}.$$

Find two choices for w, giving rise to two different decompositions of the identity $\{[w^+],[w^-]\}$, such that the family $[z^+]\odot\{[w^+],[w^-]\}\odot\{[x^+],[x^-]\}$ is consistent.

4. The toy beam splitter has a time development operator $T|mz\rangle = |(m+1)z\rangle$ for $z = a,b,c,d$ with the exception that

$$T|0a\rangle = \big(|1c\rangle + |1d\rangle\big)/\sqrt{2}, \quad T|0b\rangle = \big(-|1c\rangle + |1d\rangle\big)/\sqrt{2}.$$

(There are also exceptions to take account of periodic boundary conditions, but they are not needed in what follows.)



a) Show that the family

$$[0a]\odot\{[1c],[1d]\}\odot\{[2\bar{a}], I-[2\bar{a}]\},\ \text{where } |2\bar{a}\rangle = \big(|2c\rangle+|2d\rangle\big)/\sqrt{2},$$

with fixed initial state $[0a]$, is inconsistent.

b) Show that by replacing $[0a]$ with a different initial state $|\bar{\psi}_0\rangle$, one which lies in the subspace $[0a]+[0b]$, while leaving the projectors at later times the same as in the family in (a), one will obtain a consistent family. [Remark (i): The answer is not unique; (ii): $[0a]+[0b]$ projects onto the subspace of linear combinations of $|0a\rangle$ and $|0b\rangle$. Hint: What happens if you apply T^{-1} to $|1c\rangle$ or $|1d\rangle$?]

c) Using the family with the new initial state $|\bar{\psi}_0\rangle$ you found in (b) show that it can be made a bit more precise by replacing the rather vague $I-[2\bar{a}]$ with the sum $P+Q$ of two projectors, where P is a pure state (rank 1), and Q will occur with probability 0. [Therefore Q can be omitted from the support of the family.]