

ANNOUNCEMENT: There will be an hour exam during class the morning of Friday, Sept. 28. It will be closed book, closed notes, no pocket calculators. Bring a sharp pencil. The examination will be on the material covered in class through Wednesday, Sept. 19, and found in assignments 1 to 4. This includes the material in Chs. 1 to 3 of Townsend, and in CQT Chs. 3 (except Sec. 3.9), Ch. 4, and Ch. 5, Secs. 5.1, 5.2, 5.5, and 5.6. Also the notes “Hilbert Space Quantum Mechanics,” “Probabilities,” “Rotations and Angular Momentum” (currently being prepared) on the course web page.

COURSE WEB PAGE:

<http://quantum.phys.cmu.edu/quad/>

READING: Sources

Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed

CQT = Consistent Quantum Theory. Individual chapters at: <http://quantum.phys.cmu.edu/CQT/>

ROTAM = “Rotations and Angular Momentum” on course web page (currently under preparation).

READING: Topics

Rotations, angular momentum: Townsend Ch. 3; ROTAM.

READING AHEAD:

Unitary time development: Townsend Ch. 4; CQT Ch. 7

Composite systems: Townsend Secs 5.1 and 5.2; CQT Ch. 6

Toy models: CQT Secs. 2.5, 6.3, 7.4

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

2. [Optional, do not turn in.] Choose a coordinate system with fixed, orthogonal x , y , and z axes in three-dimensional space. Any specific rotation of a rigid body can be carried out by first rotating it by an angle α about the z axis, next a rotation of β about the x axis, and a final rotation by γ about the z axis. (Note that the first and the third rotations are about the same axis, and the axes are fixed in space, not attached to the rigid body.)

a) Find α , β and γ such that the points $(1, 0, 0)$ and $(0, 0, 1)$ are interchanged; i.e., the point on the body which is initially at distance 1 from the origin on the positive x axis finds itself after the rotation on the positive z axis, whereas a point initially on the positive z axis ends up on the positive x axis.

b) Check your answer using the rotation matrices

$$M_z(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_x(\beta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}.$$

and applying them to suitable column vectors.

c) The same rotation can be carried out as a rotation by a suitable angle about a *single* axis. What is the axis, and what is the angle?

3. Townsend Problem 3.5.

4. For $\hat{n} = (n_x, n_y, n_z)$ a (real) unit vector in three dimensions define the Pauli operator $\sigma_{\hat{n}}$ through the formula

$$\sigma_{\hat{n}} = n_x \sigma_x + n_y \sigma_y + n_z \sigma_z.$$

a) Write $\sigma_{\hat{n}}$ as a 2×2 (complex) matrix in the standard basis. Check that it is Hermitian, and that its square is equal to the identity operator, so that in these respects it resembles the well-known Pauli matrices.

b) Consider a rotation of the operator $\sigma_{\hat{n}}$ by an angle ω about the z axis, i.e.,

$$\sigma_{\hat{n}} \rightarrow \tau = e^{-i\omega\sigma_z/2} \sigma_{\hat{n}} e^{+i\omega\sigma_z/2}$$

First make a guess about what τ should be, and then calculate it by matrix multiplication to confirm your guess. You may replace your original guess by a revised version if you want to.

5. Townsend Problem 3.15: Find the eigenstates of S_x for a spin 1 particle in terms of the eigenstates of S_z . (Note that Townsend sometimes uses J and sometimes S for spin-1 angular momentum operators.) Do this by two methods: (i) Use the J_x matrix on p. 83, Eq. (3.28)—notice that J_z is diagonal in this basis—and find the eigenvectors and eigenvalues of J_x using whatever method you prefer. Good guesses are perfectly OK if checked by showing that the column vector multiplied by the matrix is a solution to the eigenvalue equation. (ii) Rotate the eigenkets of J_z by an appropriate angle around the y axis using the rotation operator supplied in Problem 3.19. Both methods should give the same answer (up to a possible phase).

6. This is Townsend's problem 3.16 done from a slightly different perspective. For a spin-1 particle we are interested in the probability that S_x has one of its three eigenvalues. To this end write (using $\hbar = 1$)

$$S_x = \sum_s s P_s; \quad s = 1, 0, -1,$$

with $\{P_s\}$ an appropriate decomposition of the identity.

a) Use the eigenstates you calculated in the previous exercise to obtain the P_s as 3×3 matrices in the S_z basis. Check that $\sum_s P_s = I$, and $\sum_s s P_s = S_x$.

b) Use $\text{Pr}(s_j) = \langle \psi | P_j | \psi \rangle$, with $|\psi\rangle$ the normalized ket corresponding to $S_z = +1$, to compute the probabilities for $S_x = 0$ or ± 1 . Check that this gives the same result as calculating probabilities using absolute squares of appropriate inner products.

7. Townsend Problem 3.6. It is not clear what sort of derivation Townsend is looking for, so use an argument based on the equality: $\langle \phi | A^\dagger | \psi \rangle = \langle \psi | A | \phi \rangle^*$ for any pair of kets $|\phi\rangle$ and $|\psi\rangle$. Assume that Eq. (3.59) is correct, and use it to obtain Eq. (3.60).