33-445 Advanced Quantum Physics Fall Semester, 2012
Assignment No. 2. Due Friday, September 7

COURSE WEB PAGE:
http://quantum.phys.cmu.edu/quad/

READING: Sources
Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed
CQT = Consistent Quantum Theory. Individual chapters at: http://quantum.phys.cmu.edu/CQT/
HSQM = "Hilbert Space Quantum Mechanics" on course web page
PROBS = "Probabilities" on course web page

READING: Topics
Hilbert space: Townsend, Chs. 1 and 2; HSQM through Sec. 2.7; CQT Ch. 3 summarizes the linear algebra in Dirac notation needed for quantum mechanics

READING AHEAD:
Decomposition of the identity: HSQM Sec. 2.9, 2.10; PROBS Sec. 2, 3; CQT Sec. 3.5
Physical properties: CQT Ch. 4
Physical variables: CQT Sec. 5.5

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course. You will find a sample answer at the end of the problem set. (Perhaps Willy is a bit too flippant, but you'll get the general idea.)

2. Let $\{|0\rangle, |1\rangle\}$ be the standard orthonormal basis for 1 qubit (for a spin-half particle, $|z^+\rangle = |0\rangle$, $|z^-\rangle = |1\rangle$) and define

$$|0'\rangle := (|0\rangle + i|1\rangle)/\sqrt{2}$$
$$|1'\rangle := (|0\rangle - i|1\rangle)/\sqrt{2}.$$

a) Show that $\{|0'\rangle, |1'\rangle\}$ is an orthonormal basis.

b) Work out the four dyads $|0'\rangle\langle 0'|$, $|0'\rangle\langle 1'|$, $|1'\rangle\langle 0'|$, and $|1'\rangle\langle 1'|$ as linear combinations of $|0\rangle\langle 0|$, $|0\rangle\langle 1|$, etc.

c) Check your answer by showing that $|0'\rangle\langle 0'| + |1'\rangle\langle 1'|$ is what you expect it to be.

3. a) Apply the dagger † operation to each of the following objects:

$$(1,\, 0,\, -i), \quad \begin{pmatrix} 1+i \\ 1-i \\ 2 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & i \\ -i & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1+i \\ 0 & -2 \end{pmatrix}$$

b) Form the matrix product AB and from it compute $(AB)^\dagger$. Compare this with the matrix product $B^\dagger A^\dagger$ of the adjoints of B and A.

4. Let $\{|0\rangle, |1\rangle\}$ be an orthonormal basis, and define another basis:

$$|\gamma_0\rangle = |0\rangle, \quad |\gamma_1\rangle = |0\rangle + |1\rangle.$$

a) We know that $\{|\gamma_0\rangle, |\gamma_1\rangle\}$ forms a basis because any $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ can be written as $\alpha'|\gamma_0\rangle + \beta'|\gamma_1\rangle$. What are α' and β' in terms of α and β?

b) Find the matrix A'_{jk} in the $\{|\gamma_0\rangle, |\gamma_1\rangle\}$ basis of the operator $A = A^\dagger = |0\rangle\langle 1| + |1\rangle\langle 0|$ using the standard definition of matrix elements:

$$A|\gamma_j\rangle = \sum_k A'_{kj}|\gamma_k\rangle.$$

(The matrix for an operator depends on the basis choice, which is why there is a prime on A'_{jk}, to distinguish it from A_{jk} in the standard basis.)

c) Construct the matrix $M_{jk} = \langle\gamma_j|A|\gamma_k\rangle$. Is it the same as A'_{jk} in (b)? Would it have been the same if $\{|\gamma_0\rangle, |\gamma_1\rangle\}$ were an orthonormal basis?

d) Construct, by specifying the coefficients B_{jk}, an operator

$$B = \sum_{jk} B_{jk}|j\rangle\langle k|$$

with the property that its matrix B'_{jk} in the $\{|\gamma_0\rangle, |\gamma_1\rangle\}$ basis is Hermitian, $(B'_{jk})^* = B'_{kj}$, but the operator B itself is *not* Hermitian.

5. a) Show that for any operator A it is the case that $\mathrm{Tr}(A^\dagger) = [\mathrm{Tr}(A)]^*$.

b) Show that $[\mathrm{Tr}(ABCD)]^*$ can be written as $\mathrm{Tr}(C^\dagger \cdots)$ for a suitable choice of $\cdots$.

6. Which of the following matrices represent projectors? Give some reason(s).

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & i/2 \\ i/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}$$

If the matrix is a projector, can you give it a physical interpretation for a spin-half particle in the form "$S_w = +1/2$ for some direction w in space" ?

Sample answer to Exercise 1 by Willy Smart.

I glanced through the stuff in Ch. 3 of Consistent Quantum Theory; everything is familiar except for the stuff on positive operators and density matrices. Well, I also have to admit that the projector stuff is a bit new. Do we *really* need to know that is meant by a decomposition of the identity? Also, is it essential that we use complex spaces? When I took the linear algebra course in my junior year we only had to use real spaces — the complex stuff was in the very next chapter that we didn't have time to cover at the end of the course.

Chapter 2 seemed elementary; I'd seen most of it before, or at least it made sense. Chapter 4 was different from what I've been exposed to previously, but not difficult. Until I came to the part on incompatible properties, which left me feeling a bit uncertain. Guess there's got to be something weird about the quantum world! Hey, prof, do you believe in Schrödinger's cat?

Problems weren't too bad, though I had a bit of difficulty with No. 3.

Complaints: Instructor tells too many jokes. Also, the pace of the course has been a bit *slow.* Until we arrived at tensor products, when it went a bit too fast.