33-445 Advanced Quantum Physics Fall Semester, 2012
Assignment No. 1. Due Friday, August 31

COURSE WEB PAGE: The following are identical
http://www.andrew.cmu.edu/course/33-445
http://quantum.phys.cmu.edu/quad/

READING: Sources
Townsend = *A Modern Approach to Quantum Mechanics*, 2d ed
CQT = Griffiths, Consistent Quantum Theory. Individual chapters are available at: http://quantum.phys.cmu.edu/CQT/
HSQM = "Hilbert Space Quantum Mechanics" on course web page
PROBS = "Probabilities" on course web page

READING: Topics
Spin half: Townsend, Chs. 1 and 2; HSQM.
Hilbert space: Townsend, Chs. 1 and 2; HSQM. CQT Ch. 3 summarizes the linear algebra in Dirac notation needed for quantum mechanics
Probabilities: CQT Ch. 5; PROBS. For classical probability theory see any introductory textbook for probability and statistics. You may find something useful at Wikipedia, though the quantum probabilities article is not very helpful.

EXERCISES:

1. Consider the following collection of kets for a spin-half particle

$$\text{(i): } |\psi_1\rangle = 2|z^+\rangle, \quad \text{(ii): } |\psi_2\rangle = |z^+\rangle - (i/2)|z^-\rangle, \quad \text{(iii): } |\psi_3\rangle = 2i|z^+\rangle + |z^-\rangle,$$
$$\text{(iv): } |\psi_4\rangle = (1/\sqrt{3})|z^+\rangle + (\sqrt{2/3})|z^-\rangle,$$

a) Write down the corresponding bra vectors as linear combinations of $\langle z^+|$ and $\langle z^-|$.

b) Which kets are normalized? For those which are not, find a normalized version $|\bar{\psi}_j\rangle$.

c) Explain why your answers to (b) are not unique.

d) Evaluate the following inner products:

$$\langle\psi_1|\psi_4\rangle, \quad \langle\psi_2|\psi_3\rangle, \quad \langle\psi_3|\psi_2\rangle, \quad \langle\psi_3|\psi_4\rangle$$

e) Consider all six pairs $\{|\psi_j\rangle, |\psi_k\rangle\}$, $j < k$, and state whether they represent the same physical property, two incompatible properties, or two mutually-exclusive properties.

f) For each $|\psi_j\rangle$ find a normalized ket which is orthogonal to it.

2. Consider for a Hilbert space of dimension $d = 3$ the kets

$$|v\rangle = \begin{pmatrix} 1 \\ 2 \\ -i \end{pmatrix}, \quad |w\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

where a, b, and c are complex numbers.

a) Write down a condition on a, b, and c which will ensure that $|w\rangle$ and $|v\rangle$ are orthogonal, $\langle w|v\rangle = 0$.

b) Find two linearly independent kets $|w_1\rangle$ and $|w_2\rangle$ such that $\langle w_1|v\rangle = \langle w_2|v\rangle = 0$.

c) [This part will not be graded, but you are strongly urged to try and find an answer.] Explain why you cannot find three linearly independent kets $|w_1\rangle$, $|w_2\rangle$, and $|w_3\rangle$ which are all orthogonal to $|v\rangle$.

3. The T-die (plural T-dice) is a solid tetrahedron with $s = 1$, 2, 3, and 4 spots on its four faces. When it is rolled it is the face next to the table, thus invisible from above, that indicates the value of s. E.g., "It came down 3" in the case where the visible faces show 1, 2, and 4 dots.

a) Alice and Bob play a game in which Bob pays Alice \$2 if $s = 2$ or 4. If $s = 1$ Alice pays Bob \$1, and if $s = 3$ Alice pays Bob \$3. Construct a random variable $W(s)$ which is the amount Alice wins ($W > 0$) or loses $W < 0$ following a roll of the T-die. Assuming equal probabilities for all four faces, find $\langle W \rangle$ and ΔW, the average and the standard deviation.

b) However, this T-die is unsymmetrical and some experimentation shows that in fact the probabilities for s are given by $p_s = 0.27$ for $s = 1$, 2, and 3. What is p_4? Again compute $\langle W \rangle$ and ΔW.

c) The game given the probabilities in (b) is no longer fair. Indicate how it can be made fair by changing just *one* of three amounts (\$1, \$2, or \$3) specified in (a). Make one of these changes, write down the new W function, call it $\bar{W}$. Check that $\langle \bar{W} \rangle = 0$, and compute $\Delta \bar{W}$.

d) After class discussion.

Thomas: "Wouldn't it be simpler to use a sample space in which there are just three possibilities, not four? These would be: $t = 1$ the same as $s = 1$, $t = 3$ the same as $s = 3$, but $t = 2$ would stand for both $s = 2$ and $s = 4$."

Ursula: "If that's possible I propose going further: a sample space with two elements: $u = 1$ would stand for both $s = 1$ and $s = 3$, and $u = 2$ for both $s = 2$ and $s = 4$."

Vincent: "Why not allow some overlap? If $s = 1$ or $s = 2$ or $s = 3$, I set $v = 1$; if $s = 2$ or $s = 4$, I set $v = 2$. Isn't my sample space as good as any of your proposals?"

William: "What I prefer is a two element sample space, where $w = 1$ is stands for $s = 1$, and $w = 2$ for $s = 2$ and $s = 4$."

It is now your turn to join the discussion. What do you think of these proposals? Are any of them satisfactory? Why or why not?

e) [This part will not be graded, but you are urged to think about it.] Before the game is played, Alice and Bob agree that it will stop as soon as the total amount that one or the other has won (or lost) exceeds \$20. Make a rough estimate of a typical number of rolls of the T-die before this agreement will bring the game to an end. [Hint. The math books tell us that if there are N independent trials the variance of the sum of the results increases proportional to N.]

4. Problem 1.3 parts (b) and (c) together with Problem 1.4 in Townsend. Check that ΔS_z and ΔS_x are zero and achieve their maximum values at the values of θ and ϕ you would expect.