33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 14
Due Friday, April 29

READING:

Two-electron atoms: Le Bellac, Sec. 14.5. A more extended treatment (from which Le Bellac has extracted some figures) is in Cohen-Tannoudji et al., Vol. 2, Ch. XIV, Complement B.

Many-electron atoms. Cohen-Tannoudji et al., Vol. 2, Ch. XIV, Complement A. There is a concise but readable treatment in the article on "Atomic Spectroscopy" in the AIP Physics Desk Reference, QC61 .A45 2003, in the reference section of the Engineering and Science library.

TOPICS (tentative):

Mon. April 25. Two-electron atoms
Wed. April 27. Many-electron atoms
Fri. April 29. Many-electron atoms

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. Suppose an electron is placed in a strong magnetic field B_0 along the z axis, as a result of which the two spin states are separated by an energy

$$\hbar\omega_0 = 2\mu_B B_0 S_z,$$

where $\mu_B = q\hbar/2m$ is the Bohr magneton, with q the electron charge and m its mass. If at some time the electron is in its excited state $S_z = +1/2$ it can decay by spontaneous emission of a photon to the ground state $S_z = -1/2$. The aim of this exercise is to calculate this decay rate using the quantized electromagnetic field and a perturbation $W = 2\mu_B \cdot \vec{B}\vec{S}$. For the quantized $\vec{B}$ use (11.84) on p. 378 of Le Bellac, setting $t = 0$ (the Schrödinger operator), and assuming that the polarization vector $\vec{e}_s(\hat{k})$ is a real unit vector perpendicular to $\vec{k}$. (Much of the discussion in Sec. 14.3.4 of Le Bellac carries over to the present case, including $e^{i\vec{k}\cdot\vec{r}} \approx 1$.)

a) Suppose the spontaneous photon is emitted with $\vec{k}$ in the x,z plane, with $\vec{k}$ making an angle θ with the z axis. Work out the absolute square of the perturbation matrix element assuming that (i) $\vec{e}_s$ lies in the x,z plane, and (ii) that it is perpendicular to the x,z plane. (In thinking about the physics it is helpful to remember that $\vec{e}_s$ corresponds to the direction of the electric field, so $\hat{k} \times \vec{e}_s$ gives the direction of the magnetic field.)

b) Find a formula for the spontaneous transition rate per unit solid angle as a function of θ and ϕ if the polarization of the outgoing photon is ignored, and then integrate this to find the total transition rate. Compare the result in terms of its dependence on signficant physical parameters (not numerical factors such as $4!/\pi$) with Le Bellac's result in Sec. 14.3.4 for the decay of $2p$ to $2s$ in hydrogen.

c) What is the numerical value of the transition rate for an electron in a field B_0 of 100 Tesla?

3. Cohen-Tannoudji Ch. XIV, Complement D, No. 1, worded a little differently.

Suppose that the one-particle states (orbitals) corresponding to a particular (approximate) one-particle Hamiltonian have energies of 0, $\hbar\omega_0$ and $2\hbar\omega_0$. For electrons each orbital is 2-fold degenerate due to the spin degeneracy. Let the full Hamiltonian be the sum of the one-particle Hamiltonians (i.e., ignore interactions).

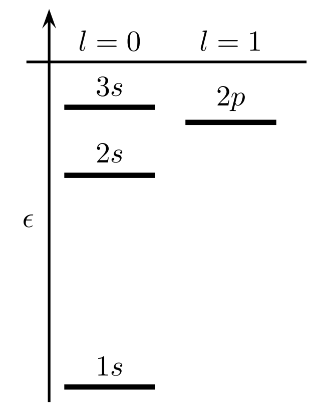
a) Given three electrons, what are the energy levels (of the full Hamiltonian) and the degeneracy of each level?

b) Replace the three electrons with three identical bosons of spin 0, assuming in this case that each of the orbitals is nondegenerate. Again find the energy levels and the degeneracy of each.

4. A neutral lithium (Li) atom contains three electrons. In a central potential approximation the one-particle orbitals have (negative) energies

$$\epsilon(1s) < \epsilon(2s) < \epsilon(2p) < \epsilon(3s) < \cdots$$

as indicated in the figure. Because of electron spin, there are actually two $1s$ orbitals: $1s+$ and $1s-$, corresponding to $S_z = +1/2, -1/2$. Similarly there are two $2s$ and two $3s$ orbitals, and six $2p$ orbitals.



What is the configuration (list of occupied orbitals) and what is the energy of (i) the ground state and (ii) the first excited state of Li? What is the degeneracy of each of these states? Give a brief explanation of your reasoning. Energies should be expressed in terms of $\epsilon(1s)$, etc. Ignore corrections to the energy from effects not included in the central potential approximation of independent orbitals; in particular, ignore spin-orbit effects. Also ignore nuclear spin.