33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 12
Due Friday, April 15 by 2:00 pm

Please put your completed assignment under the door of WEH 6309.

READING:

One-electron atoms: fine structure, Zeeman effect, hyperfine structure. Le Bellac, Sec. 14.2. These topics are discussed in many textbooks.

Interaction of atoms with electromagnetic fields (expect to get to this topic the week of April 18): Le Bellac, Sec. 14.3. Secs. 14.3.1 to 14.3.3 are found in many textbooks, whereas 14.3.4, spontaneous emission uses a quantized electromagnetic field.

We will not be taking up Le Bellac Sec. 14.4 (trapping of atoms), but we will take up Sec. 14.5 (two-electron atom).

TOPICS (tentative):

Mon. April 11. Zeeman effect; begin hyperfine interaction
Wed. April 13. Hyperfine interaction both in zero and finite magnetic field
Fri. April 15. Watch the buggies, but remember the problem assignment is due by 2:00 pm.

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. Spin orbit coupling is observed in atoms with two valence electrons, whose spins can add up to form an $s = 1$ (triplet) state. Assume a spin-orbit interaction $W = \zeta \vec{L} \cdot \vec{S}$, with $\vec{L}$ and $\vec{S}$ dimensionless operators (i.e., expressed in units of $\hbar$), ζ a constant with dimensions of energy, and an orbital angular momentum quantum number is $l = 1$.

a) Work out how the spin-orbit interaction splits the 9-fold degenerate level into sublevels for different j (total angular momentum) quantum numbers. Which sublevel is highest assuming that $\zeta > 0$? Check that your answer agrees with $\mathrm{Tr}(W) = 0$.

b) It is asserted in an article in the *Encyclopedia of Physics*, 3d ed., "The ratio of separations between adjacent components of a fine structure multiplet, $(j, j-1)$ and $(j-1, j-2)$ is $j/(j-1)$." Derive this result assuming that $W = \zeta \vec{L} \cdot \vec{S}$, for general values of s and l, and check that it is satisfied by your results in (a). (According to the same source, this ratio rule is obeyed reasonably well by intermediate weight elements, but not by very light or very heavy elements.)

3. The proton and deuteron have spin 1/2 and 1, and their magnetic moments are 2.7923 and 0.857 nuclear magnetons, respectively.

a) Make an estimate of the hyperfine splitting in deuterium starting with the value in hydrogen and then using the fact that the spins and magnetic moments are different. (Ignore other effects).

b) In the presence of a magnetic field B in the z direction there is a Zeeman effect: the hyperfine levels of hydrogen split. Assume an interaction of the form

$$W = A \vec{I} \cdot \vec{S} - 2\mu_B B S_z.$$

(In particular ignore the interaction of the B field with the nucleus), where $\vec{I}$ and $\vec{S}$ are the (dimensionless) nuclear and electron spin operators, and μ_B is the Bohr magneton. Write W as a 4×4 matrix using the basis states $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, where the first argument refers to the proton and the second to the electron, and find its eigenvalues. (This reduces to the problem of diagonalizing a 2×2 matrix.) Make a schematic (or more sophisticated, if you prefer) plot of the energies as a function of B. (If you need hints or get stuck, there is an exhaustive discussion in Ch. XII of Cohen-Tannoudji et al.)

c) Now consider the corresponding Zeeman effect in deuterium, but only when the magnetic field B is small. Indicate how the levels split, and compare the rate of splitting as a function of B with what happens for a single electron spin in a magnetic field.

4. (Related to Exercise 14.6.3 in Le Bellac.) The muon is a "heavy electron" with a mass $m_\mu/m_e = \rho \approx 207$. In terms of ρ (consider only the dominant order), what do you expect to be the ratio of the muonic hydrogen binding energy to that of electronic hydrogen? In a similar way compare fine structure splitting and hyperfine structure splitting for these two types of atom. Clearly indicate your reasoning.