33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 11
Due Friday, April 8

READING:

Perturbation theory and variational method: Le Bellac Sec. 14.1, which is fairly compact. These topics are treated in all textbooks; e.g., Cohen-Tannoudji et al. For a more advanced treatment of perturbation theory including the method of resolvents, see Messiah, *Quantum Mechanics*, Vol. II, Ch. XVI. Some handwritten notes, “Perturbation Theory for a Nondegenerte Level,” that correspond to the lecture given in class are (or shortly will be) on the course web site.

One-electron atoms: fine structure, Zeeman effect, hyperfine structure. Le Bellac, Sec. 14.2. These topics are discussed in many textbooks

TOPICS (tentative):

Mon. April 4. Perturbation theory
Wed. April 6. Variational method. Use of symmetries
Fri. April 8. One-electron atoms

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. a) Find formulas for the eigenvalues E^+ and E^- of the 2×2 Hermitian matrix

$$H = \begin{pmatrix} a & b \\ b^* & c \end{pmatrix}.$$

b) Assume that

$$H = H_0 + H_1 = \begin{pmatrix} 0 & 0 \\ 0 & \Gamma \end{pmatrix} + \begin{pmatrix} a & b \\ b^* & \gamma \end{pmatrix},$$

where a, b and γ are small quantities of order λ, with λ small in comparison to the positive energy Γ. Expand E^+ and E^- to second order in small quantities. Compare with the standard formulas given by perturbation theory.

c) Find the first-order correction $|\varphi_1\rangle$, as a column vector, to the eigenstate which to zeroth order is $|\varphi_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and check the formula $E_2 = \langle\varphi_0|H_1|\varphi_1\rangle$ for the second-order correction to the energy.

3. From Cohen-Tannoudji et al., Ch. XI, No. 1 on p. 1200 of Vol. 2 (slight modification of wording).

A particle of mass m is placed in an infinite one-dimensional well $V(x)$ of width a,

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le a, \\ +\infty & \text{elsewhere.} \end{cases}$$

It is subject to a perturbation

$$W(x) = aw_0\delta(x - a/2),$$

where w_0 is a real constant with the dimensions of energy.

a) Calculate, to first order in w_0, the modifications induced by $W(x)$ in the energy levels of the particle.

b) Actually, the problem is exactly soluble. Setting $k = \sqrt{2mE/\hbar^2}$ show that the possible values of the energy are given by one of the two equations $\sin(ka/2) = 0$ or $\tan(ka/2) = -\hbar^2 k/maw_0$. Discuss how these energies depend on the sign and size of w_0. Show that as $|w_0|$ tends to zero the result agrees with (a).

4. Use the functions (the expressions below are not normalized)

$$\psi_0(x) = \begin{cases} \cos kx & \text{for } -\pi/2 \le kx \le \pi/2, \\ 0 & \text{elsewhere,} \end{cases}$$

$$\psi_1(x) = \begin{cases} \sin kx & \text{for } -\pi \le kx \le \pi, \\ 0 & \text{elsewhere,} \end{cases}$$

with k is a variational parameter, to obtain estimates which are upper bounds on the ground and first excited state energies of a one-dimensional harmonic oscillator. Indicate briefly how you know that $\psi_1(x)$ yields an upper bound on the first excited state. [Hint. The integrals can be done in closed form. You may prefer to use a calculus program.]