33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 10 (Revised)
Due Friday, April 1

ANNOUNCEMENT. HOUR EXAM. The second hour exam will be on Monday, April 4, at 6:00 pm in WEH 7316 (same room as lectures). The exam will be closed book, closed notes. Bring a pencil or pen. You will be responsible for material up to and including the lecture of March 21 (but *not* including material on identical particles), and through Asn. 9 (again, not including identical particles). The emphasis will be on material covered since the last hour exam: field theory (including electromagnetic field) and scattering. This means Assignments 7, 8, and 9.

READING:

Identical particles. Le Bellac Secs. 13.1, 13.2. Other quantum textbooks, such as Cohen-Tannoudji et al. (Vol. II, Ch. XIV), Sakurai, Shankar, discuss this topic. Supplementary notes "Systems of Identical Particles" on course web page.

Creation and annihilation operators for fermions and bosons. Supplementary notes "Creation and Annihilation Operators" on course web page.

Scattering of two identical particles. The treatment in Le Bellac is very compact. Taylor, *Scattering Theory*, Ch. 22, sections *a* and *b* is (for the most part) accessible without having mastered the earlier chapters. (Supplementary notes may later be available on course web page.)

Time-independent approximation methods. Le Bellac Sec. 14.1

TOPICS (tentative):

Mon. March 28. Creation and annihilation operators. Scattering of identical particles
Wed. March 30. Scattering of identical particles. Time-independent approximation methods
Fri. April 1. Time-independent approximation methods

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. For a spin half particle, let $\alpha(\vec{r})$ and $\beta(\vec{r})$ be mutually orthogonal and normalized spatial functions, and $\chi_+(\sigma)$ and $\chi_-(\sigma)$ spin states corresponding to $S_z = \pm 1/2$, and use the notation

$$\alpha_+(q_1) = \alpha(\vec{r}_1)\chi_+(\sigma_1),$$

etc.

a) Write out the wave function $\langle q_1 q_2 | \alpha_+ \alpha_-, A\rangle$, where A signifies that it is antisymmetrical under exchange of the particles, as a function of $\vec{r}_1, \vec{r}_2, \sigma_1, \sigma_2$, and show that it is symmetrical under interchange of the space coordinates, and antisymmetrical under the interchange of the spin coordinates. What is the value of the total spin quantum number S?

b) Write out the wave function $\langle q_1 q_2 | \alpha_+ \beta_+, A\rangle$ and discuss its symmetry under exchange of space coordinates and and of spin coordinates. What is the value of S?

c) Next write out $\langle q_1 q_2 | \alpha_+ \beta_-, A\rangle$ and $\langle q_1 q_2 | \alpha_- \beta_+, A\rangle$. Look for linear combinations which have have some symmetry under interchange of space or interchange of spin coordinates, and indicate the corresponding values of S.

3. Consider the scattering of deuterons by deuterons at energies which are low enough that the interaction can be considered as spin-independent. Calculate the differential cross section for scattering at 90° in the center of mass, expressing your answer in terms of the amplitude $f(\theta = \pi/2)$ for almost-identical particles, assuming:

i) A spin-polarized incident beam and target, with the spins parallel (say $M_S = 1$ for both projective and target);

ii) an unpolarized beam and target (random orientation of spin).

Note that a deuteron is a spin-one boson, and the $S = 0$ and $S = 2$ spin states for a pair of deuterons are symmetrical, whereas the $S = 1$ states are antisymmetrical, under interchange of the particles.

4. a) Suppose that boson creation operators $\{b_j^\dagger\}$ are defined using a collection $\{|\alpha_j\rangle\}$ of orthonormal single particle orbitals, i.e., $|\alpha_j\rangle = b_j^\dagger|\emptyset\rangle$, and let $\{\hat{b}_k^\dagger\}$ be similarly defined using an alternative orthonormal set $\{|\hat{\alpha}_k\rangle\}$. Express the $\hat{b}_k^\dagger$ as linear combinations of the $b_j^\dagger$, and use that relationship to show that if the $\{b_j^\dagger\}$ and their adjoints satisfy the commutation relations appropriate to boson creation and annihilation operators, the commutation relations are also satisfied by the $\{\hat{b}_k^\dagger\}$.

b) Same exercise for identical fermions: the $\{f_j^\dagger\}$ and $\{\hat{f}_k^\dagger\}$ correspond to single-particle states $\{|\alpha_j\rangle\}$ and $\{|\hat{\alpha}_k\rangle\}$, and anticommutation relations replace commutation relations.

5. DO NOT TURN IN. From an hour exam given in a previous course.

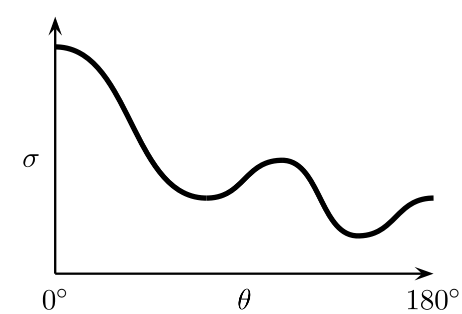
The total scattering cross section σ_T for scattering from a spherically symmetrical potential can be written in the form

$$\sigma_T = \sigma_0 + \sigma_1 + \sigma_2 + \cdots ,$$

where σ_j is the contribution from the j'th partial wave.

a) Derive an expression relating σ_j to the corresponding phase shift δ_j (see formulas at the end), and indicate why the differential cross section $\sigma(\Omega)$ (or $d\sigma/d\Omega$) *cannot* be written as a sum $\sigma_0(\Omega) + \sigma_1(\Omega) + \sigma_2(\Omega) + \cdots$ of contributions from individual partial waves.

b) A cross section is observed to have two minima as a function of θ, as in the sketch. Assume the phase shifts δ_l are extremely small for all $l > l_0$, where l_0 is some integer. What is the smallest possible value of l_0? Give some explanation.



6. DO NOT TURN IN. From an hour exam in a previous course

a) Consider scattering from a class of potentials $cV(\mathbf{r})$, where $V(\mathbf{r})$ is fixed and c is a (possibly negative) real number. How does Born approximation for the differential cross section at a particular angle and for a particular energy depend on c?

b) Let $V(\mathbf{r})$ be a spherically-symmetrical potential. Show that in the Born approximation the differential scattering cross section at 120° in the center of mass is the same as that at 60° for a different incident energy (in the center of mass). How are the two energies related to each other? Indicate at which point in your argument you use the fact that $V(\mathbf{r})$ is spherically symmetrical. Assume nonrelativistic particles (energy proportional to momentum squared).