33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 9
Due Friday, March 25

ANNOUNCEMENT. HOUR EXAM. The second hour exam will be on Monday, April 4, at 6:00 pm in WEH 7316 (same room as lectures). The exam will be closed book, closed notes. Bring a pencil or pen.

READING:

Born approximation. Le Bellac Sec. 12.4.1. This is a very compact discussion. For more details you may wish to consult Cohen-Tannoudji et al. Vol. II, Ch. VIII, Secs. B.3 and B.4, or some other quantum textbook.

Identical particles. Le Bellac Secs. 13.1, 13.2. Cohen-Tannoudji, Vol. II, Ch. XIV gives a more extended discussion. Supplementary notes on course web page (currently under preparation).

TOPICS (tentative):

Mon. March 21. Scattering in Born approximation. Examples
Wed. March 23. Identical particles. General formalism
Fri. March 25. Identical particles. Examples

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. a) Apply the Born approximation (in three dimensions) to a potential which is a delta function on a sphere of radius a,

$$V(\vec{r}) = V_0\delta(r-a),$$

and plot the differential cross section as a function of qa, $\vec{q} = \vec{k}_s - \vec{k}_0$, out to $qa \approx 8$.

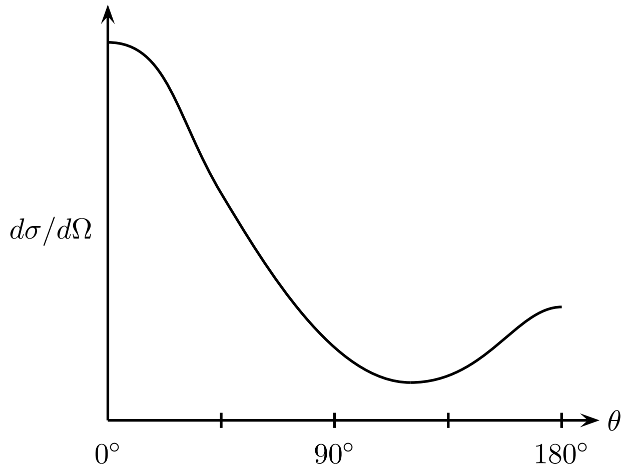
b) Evaluate the cross section in Born approximation for a square-well potential

$$V(\vec{r}) = \begin{cases} V_0 & \text{for } r < a, \\ 0 & \text{for } r > a. \end{cases}$$

How does the separation between successive minima as a function of qa compare with what you found in part (a)?

c) Consider the scattering of protons by a carbon nucleus. If the Coulomb potential is ignored, the nuclear force can be approximated by a square well potential with $a \approx 2 \times 10^{-15}$m. Estimate the proton energy which would be needed in order to see two minima in the differential cross section as a function of angle.

3. The differential cross section $\sigma'(\theta) = d\sigma/d\Omega$ as a function of angle θ in the center of mass is measured at several values of the energy E for a particle scattering from a target with a short-ranged spherically symmetrical potential. At low energies $\sigma'(\theta)$ is independent of angle, but as the energy increases it develops a peak in the forward direction, and at the energy E_1 it has a minimum near $\theta = 120°$, as indicated in the sketch.



a) What is the minimum number of partial waves required to produce a cross section of this general form, and which partial waves are involved? Give reasons for your answer.

b) Indicate how one can make a rough estimate of the range R of the potential in terms of E_1 and the reduced mass μ.

c) As the energy increases, the minimum moves to smaller values of θ. Use the Born approximation to estimate the angle at which the minimum will occur if the energy is increased to $2E_1$. You do not have to obtain an explicit result, but your procedure should be clear. (Sketches are helpful.)

4. In answering this question, *ignore* the usual connection between spin and statistics; i.e., assume that spin zero particles can be fermions or bosons, and likewise particles with spin one half. You can label states using F (fermions) and B (bosons), or A (antisymmetric) and S (symmetric).

Consider spinless quantum particles in a one-dimensional square well in which $V(x)$ is infinite except for $-a < x < a$, where it is zero. Let the orbital $\phi_n(x)$ have energy ϵ_n, with $n = 1, 2, 3, \ldots$ in order of increasing energy. Express energies as multiples of ϵ_1.

a) Find the lowest three energy levels of a system of $N = 2$ noninteracting particles, and the corresponding configurations, i.e., the number of particles in each orbital. Do not bother to work out the wave functions. Assume that

i) the particles are identical bosons,

ii) the particles are identical fermions.

Indicate whether the levels are degenerate or non-degenerate; in the case of a degenerate level, find the degeneracy and write down all of the corresponding configurations.

b) Same question for $N = 3$.

c) Same question for $N = 2$ identical spin one half particles; use $+$ and $-$ to indicate spin states, and assume the single particle energy does not depend on the spin.