33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 8
Due Friday, March 18

READING:

Scattering: LB Sec. 12.1 (amplitude, cross section).

Sec. 12.2 (partial waves) including 12.2.1 and 12.2.2 (ignore part on effective range), but not 12.2.3 or 12.2.4.

Sec. 12.3.1 (optical theorem) but not 12.3.2 (optical potential)

Sec. 12.4.1 (Born approximation)

While Le Bellac mentions and uses the S matrix or scattering operator, he never defines it properly for three-dimensional scattering. The lecture will provide a simple introduction. Some handwritten notes on the S matrix are available at the course web site. An excellent but more lengthy introduction will be found in Ch. 2 of John Taylor, *Scattering Theory*, which I have asked the librarian to place on reserve.

TOPICS (tentative):

Mon. March 14. Partial waves, phase shifts, scattering matrix

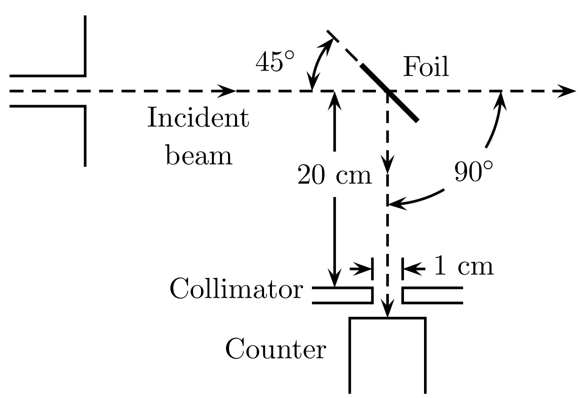
Wed. March 16. Examples of scattering

Fri. March 18. Born series and Born approximation

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. A counter is placed behind a circular collimator of diameter 1.00 cm located 20.0 cm from a scattering target which is an aluminum foil mounted at 45° to the incident beam direction, see figure. The foil has a thickness such that 1 cm^2 weighs 5.00×10^{-3}g.



Suppose that the beam consists of 12 MeV protons with a current of 1.00×10^{-7} amps, and that the counter registers 250 counts/s. Calculate the differential cross section $d\sigma/d\Omega$ (in the laboratory) for scattering from the aluminum nucleus, in cm^2/steradian, at an angle of 90° (as shown in the figure).

3. Find the $l = 0$ (s-wave) phase shift for scattering from a potential

$$V(r) = \begin{cases} V_0 > 0 & \text{for } r < a, \\ 0 & \text{for } r > a. \end{cases}$$

assuming that the energy E of the incident particle is less than V_0. What is its behavior as k tends to zero? Compare this with the result for a hard sphere. Find an explicit expression for σ_0, the $l = 0$ contribution to the total cross section, and show that it has the expected form as k tends to zero.

4. Suppose that the differential cross section for a certain potential is determined entirely by the $l = 0$ and $l = 1$ phase shifts.

a) Work out expressions for the differential cross sections as a function of angle θ in the following cases, and then plot the results. Do not worry about the overall normalization, but obtain the correct angular dependence.

i) $\delta_0 = 0$, $\delta_1 = 30°$
ii) $\delta_0 = 60°$, $\delta_1 = 30°$
iii) $\delta_0 = 60°$, $\delta_1 = -30°$

b) Where as a function of the parameters $0 \leq \delta_0 \leq \pi$, $0 \leq \delta_1 \leq \pi$ (you can think of them as forming a square) will the differential cross section $d\sigma/d\Omega$ as a function of θ be:

i) Symmetrical about $\theta = 90°$?
ii) Peaked in the forward direction: larger near $\theta = 0°$ than near $\theta = 180°$?
iii) Peaked in the backward direction: larger near $\theta = 180°$ than near $\theta = 0°$?

5. Consider a potential

$$V(r) = V_0 \delta(r - R),$$

where V_0 can be positive or negative, $R > 0$, and $\delta(r - R)$ is a Dirac delta function.

a) Find an expression which determines the $l = 0$ phase shift δ_0 in terms of the dimensionless quantities

$$W := 2mV_0 R/\hbar^2, \quad \rho := kR,$$

where the energy is $\hbar^2 k^2/2m$. [Hint: A moderately simple formula involves $\cot(\rho + \delta_0)$ and $\cot(\rho)$.]

b) Find an expression in terms of $\hat{\rho} = \kappa R$ which determines the energy $E = -\hbar^2\kappa^2/2m$ of the $l = 0$ bound state (there is only one) when W is sufficiently negative.

c) The scattering matrix element for $l = 0$ is given by

$$S_0 = e^{2i\delta_0} = \frac{1 - i(W/2\rho)(1 - e^{-2i\rho})}{1 - i(W/2\rho)(e^{2i\rho} - 1)}$$

Show that if both W and ρ are real, then $|S_0| = 1$ as expected from unitarity. Next show that the denominator vanishes (indicating that S_0 has a pole) at the (complex) value of ρ you would expect given the bound state determined in (b).