33-756 Quantum Mechanics II Spring Semester, 2011
Revised Assignment No. 7
Due **Wednesday** March 2

READING:

Electromagnetic field quantization: LB Sec. 11.3.3. We will not take up Sec. 11.3.4.

Supplementary handwritten notes on course web page: "Quantization of Stretched String", "Notes on Coaxial Resonator" may be helpful in working out the exercises.

Scattering: LB Sec. 12.1 (amplitude, cross section), 12.2.1 (partial waves).

TOPICS (tentative):

Mon. Feb.. 28: Finish electromagnetic field. Begin scattering.

Wed. March 2. Scattering. Begin partial waves.

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. Le Bellac exercise 4.4.7. (This discussion of Heisenberg operators generalizes the one one on p. 114. Note that Le Bellac's $U(t, t_0)$ is what in class was denoted by $T(t, t_0)$.)

3. For the chain of atoms in Le Bellac Sec. 11.3.1 find an expression for $\langle Q_n Q_{n'} \rangle$ in the quantum ground state. (These are the quantum operators; in class we used q_n and $q_{n'}$.) Write your answer as a sum over the modes k, with $k = 0$ omitted (since it is infinite); do not try and evaluate the sum in closed form. [Hint: The average only depends on the difference $|n' - n|$.] Show that even with the $k = 0$ term omitted there is something problematical in the case $n' = n$ if one lets L or N tend to infinity with the distance l between adjacent atoms held fixed. [Hint: Use the approximation $\omega_k \approx c_s k$ when $k > 0$ is small, see Le Bellac's (11.46), and ask what will happen to the sum as $L = Nl \to \infty$.]

4. For the stretched string of length L discussed in class (see notes posted on the course web page) we chose "normalized" normal modes $f_n(x) = \sqrt{2/L}\,\sin(n\pi x/L)$. Suppose a different normalization had been used, e.g., $g_n(x) = \sin(n\pi x/L)$, with

$$\varphi(x, t) = \sum_n c_n(t) g_n(x)$$

for the classical string. Would the quantization process in which the c_n is replaced with a suitable linear combination of a_n and $a_n^\dagger$, satisfying the usual commutation relation $[a_n, a_n^\dagger] = 1$, lead to a different expression for the final quantum operator $\varphi(x)$ (or $\Phi(x)$) in terms of creation and annihilation operators?

5. Consider a coaxial resonator with inner and outer radii of ρ_1 and ρ_2, shorted out with end plates at $z = 0$ and $z = L$. In what follows consider only the simple modes for which in cylindrical coordinates (r, ϕ, z) the vector potential $\vec{A}$ is purely radial and has the form

$$A_{nr}(r, \phi, z) = [b_n(t)/r] \sin k_n z$$

for the n'th mode, $n = 1, 2, 3 \ldots$, in the region $\rho_1 \leq r \leq \rho_2$ and $0 \leq z \leq L$. Here $k_n = n\pi/L = \omega_n/c$, where ω_n is the corresponding angular frequency, and c is the speed of light. Assume the scalar potential V is zero.

a) Assuming classical electromagnetism, find expressions for the electric and magnetic fields, $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$ for the n'th mode, and then an expression for the energy H_n in this mode, in terms of b_n and its time derivative $\dot{b}_n$. Try and write things in terms of $G := 2\pi L \ln(\rho_2/\rho_1)$, as this will make the exercise easier to grade.

b) Assume $\rho_1 = 1$ mm and $\rho_2 = 2.7$ mm ($\ln \rho_2/\rho_1 = 1$) and $L = 1$ cm. Suppose that the fundamental mode $n = 1$ is excited in such a way that the maximum (in space and time) electric field E_0 inside the resonator is 1 V/m. Assuming that this corresponds to a quantum coherent state for this mode, what is the average number of photons present, and what is the standard deviation for this number?

c) In order to "see" quantum effects in such a resonator one needs to cool it to a temperature T such that $k_B T$ is less than $\hbar\omega$. What temperature is involved? (It is not out of the range of modern cooling techniques.)

d) Find expressions for the electric $\vec{E}(\vec{r})$ and magnetic field $\vec{B}(\vec{r})$ quantum (Schrödinger) operators as sums involving creation and annhiliation operators for the normal modes under consideration. Comment on how these compare with the expressions on p. 378 of Le Bellac, where you can set $t = 0$ to turn the Heisenberg operators E_H and B_H into the corresponding Schrödinger operators.