33-756 Quantum Mechanics II Spring Semester, 2011
Assignment No. 6
Due Friday, Feb. 18

ANNOUNCEMENT. HOUR EXAM. The first hour exam will be on Monday, Feb. 21 at 6:00 pm in WEH 7316 (same room as lectures). The exam will be closed book, closed notes. Bring a pencil or pen. You will be responsible for the material covered in the lectures up to and including Feb. 11 (coherent states), and on problem assignments 1 through 6.

READING:

Coherent states: LB Sec. 11.2. There is a much more extensive treatment in Cohen-Tannoudji et al. Quantum Mechanics, Vol. I, Complement G to Ch. V.
Quantization of phonons: LB Sec. 11.3.1
Electromagnetic field quantization: LB Sec. 11.3.3
Landau levels: LB Sec. 11.4.2

TOPICS (tentative):

Mon. Feb. 14. Sound waves and phonons
Wed. Feb. 16. Quantization of electromagnetic field
Fr. Feb. 18. Motion in a uniform magnetic field (Landau levels)

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include comments about the lectures, complaints about the course, etc.

2. Using their connection with harmonic oscillator states, show that Hermite polynomials obey an orthogonality relation

$$\int_{-\infty}^{\infty} w(u) H_m(u) H_n(u)\, du = \delta_{mn} f_n,$$

for a suitable weight function $w(u)$, which you should specify, and an appropriate f_n.

3. Let $|n\rangle$ be number states for a harmonic oscillator.

a) Let $p \geq 0$ and $q \geq 0$ be nonnegative integers. Find conditions on m and n such that the matrix elements $\langle m|a^p (a^\dagger)^q|n\rangle$ are nonzero, and give expressions in terms of n, p, and q for the nonzero values. Do the same for the matrix elements $\langle m|(a^\dagger)^q a^p|n\rangle$.

b) Let $w = d/du$, where u is the dimensionless position of the oscillator particle. Express w in terms of a and $a^\dagger$, and use the resulting expression to evaluate the matrix elements $\langle m|w^3|n = 2\rangle$ for different values of m. [Hint: Because a and $a^\dagger$ do not commute, w^3 involves a large number of terms. Use commutation relations to simplify it.]

4. a) Show that (provide a plausible argument; a mathematically rigorous proof is not required) the operator

$$D(z) = \exp(-z^* a + z a^\dagger)$$

is unitary.

b) Show that the coherent state $|z\rangle_c$ is equal to $D(z)|0\rangle$. [Hint. See part 4 of Le Bellac's exercise 11.5.3.]

c) Let w be any complex number; show that $D(z)$ applied to the coherent state $|w\rangle_c$ yields another coherent state up to some constant (which you should evaluate).

5. Consider a quantum harmonic oscillator in two dimensions with Hamiltonian

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2} m\omega^2 \left(x^2 + y^2\right).$$

Show how to construct a coherent state whose intuitive interpretation is that of a particle moving in a clockwise (as seen from above) circle in the x, y plane, starting at $t = 0$ with $\langle x\rangle = r_0 > 0$ and $\langle y\rangle = 0$, and then a short time later $\langle y\rangle < 0$. Use dimensionless distances $u = x/\xi$, $v = y/\xi$, with $\xi = \sqrt{\hbar/m\omega}$, and use complex numbers α and β to label the coherent states corresponding to the x and y motion. (Using $z = x + iy$ to label coherent states would cause confusion with the x and y mechanical degrees of freedom.) Thus the wave packet $\langle u|\alpha\rangle$ (or $\chi(u, \alpha)$) denotes the coherent state corresponding to the complex number α. Work out the unitary time evolution of the wave packet $\psi(u, v, t)$. [Hint: It can be written as a tensor product on $\mathcal{H}_x \otimes \mathcal{H}_y$.] Indicate why you think the particle is moving (in some average sense) on the expected orbit.

Explain how you can in a similar way achieve an elliptical orbit, or a circular orbit moving in the opposite (counterclockwise) sense.