33-658, 758 Quantum Computation and Information Spring Semester, 2014
Assignment No. 1. Due Tuesday, January 21

In the future all assignments will be posted at the COURSE WEB SITE: http://quantum.phys.cmu.edu/QCQI/

READING:

QCQI = Nielsen and Chuang, Quantum Computation and Quantum Information
CQT = Griffiths, Consistent Quantum Theory
HSQM = "Hilbert Space Quantum Mechanics" Course web site

History: QCQI Sec. 1.1

Linear algebra: Review using your favorite book. What you need to know is in:
CQT Secs. 3.1 to 3.7; QCQI Sec. 2.1

Introduction to quantum mechanics:
CQT Ch. 2 and Secs. 4.1 and 4.2; QCQI Secs. 1.2, 2.2; HSQM Secs. 1,2,3

Composite systems:
CQT Ch. 6; QCQI Secs. 1.2.1, 2.1.7; HSQM Sec. 4

READING AHEAD:

Unitary dynamics and quantum circuits:
CQT Ch. 7; QCQI Secs. 1.3.1, 1.3.2, 1.3.4, 1.3.6; 2.2.2; 4.2, 4.3

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course. You will find a sample at the end of the problem set.

2. Let

$$|\chi\rangle = |0\rangle - i|1\rangle, \quad |\omega\rangle = \frac{(1+i)}{\sqrt{2}}|0\rangle + |1\rangle$$

a) Find the matrix of the operator $|\chi\rangle\langle\omega|$ in the standard basis.
b) Check that $\mathrm{Tr}(|\chi\rangle\langle\omega|) = \langle\omega|\chi\rangle$.
c) A *normal* operator N is one such that $NN^\dagger = N^\dagger N$. How must $|\psi\rangle$ be related to $|\phi\rangle$ so that $|\psi\rangle\langle\phi|$ is a normal operator?

3. Which of the following matrices represent projectors? Give some reason.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & i/2 \\ i/2 & 1/2 \end{pmatrix}, \quad \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}$$

If the matrix is a projector, can you give it a physical interpretation in the form: "the spin is in the direction ... on the Bloch sphere"?

4. Find the eigenvalues a_0 and a_1 of

$$A = \begin{pmatrix} 1/2 & -i \\ i & 1/2 \end{pmatrix},$$

and find $|\psi_0\rangle$ and $|\psi_1\rangle$ (using the standard basis) such that

$$A = a_0|\psi_0\rangle\langle\psi_0| + a_1|\psi_1\rangle\langle\psi_1|.$$

Check your answer by calculating $|\psi_0\rangle\langle\psi_0|$ and $|\psi_1\rangle\langle\psi_1|$ as 2×2 matrices, and showing that the last equation is satisfied.

5. Let w be the direction in space corresponding to a vector with components $w_x = -w_y = w_z = 1/\sqrt{3}$. What are the angles θ_0 (colatitude) and ϕ_0 (azimuth), in degrees, corresponding to this in polar coordinates? Write kets corresponding to $S_w = +1/2$ and $S_w = -1/2$ in the form $|0\rangle + \gamma|1\rangle$, i.e., find the complex number γ for the two cases. Check that the two kets are orthogonal, and that they correspond to antipodes ($\theta_1 = 180° - \theta_0,\ \phi_1 = \phi_0 + 180°$) on the Bloch sphere. Then find the corresponding normalized kets.

6. Show that on the tensor product of two qubits the state

$$|\psi\rangle = |w^+w^-\rangle - |w^-w^+\rangle$$

is equal to $|01\rangle - |10\rangle$, independent of the direction w on the Bloch sphere. Remark: If you make a choice other than the "standard" phases in defining $|w^+\rangle$ and $|w^-\rangle$, the the result will be $|01\rangle - |10\rangle$ up to a phase factor, which does not change the physical interpretation. (The state $|\psi\rangle$ is known as a "spin singlet state". Bohm used it to discuss the Einstein-Podolsky-Rosen paradox, so it is sometimes called an "EPR pair". It is one of the so-called "Bell states.")

7. For two qubits (spin-half particles) a and b, let

$$|\psi\rangle = \Big(|0\rangle_a|0\rangle_b + |1\rangle_a|1\rangle_b\Big)/\sqrt{2}.$$

a) Show that the properties $[\psi] = |\psi\rangle\langle\psi|$ and $P = [0]_a$ are incompatible by evaluating the commutator $[\psi]P - P[\psi]$. Remark: As usual, $[0]_a$ should be interpreted as $[0]_a \otimes I_b$.

b) Show that $[\psi]$ is incompatible with *any* nontrivial property A of spin a, where the trivial properties are I_a and 0. [Hint 1. Any nontrivial property will be represented by a projector $A = [w]_a \otimes I_b$, where $[w] = |w\rangle\langle w|$ corresponds on the Bloch sphere to some direction w in space. Hint 2. Any operator R on the two-qubit Hilbert space can be written in the form

$$R = S_a^{(00)} \otimes [0]_b + S_a^{(11)} \otimes [1]_b + S_a^{(01)} \otimes |0\rangle\langle 1|_b + S_a^{(10)} \otimes |1\rangle\langle 0|_b,$$

where the operators $S_a^{(jk)}$ are uniquely determined by R.]

c) Show that there is no one-dimensional product property (rank one projector) $P = A_1 \otimes B_1$, where A_1 and B_1 are one-dimensional projectors, such that a system in state $|\psi\rangle$ has property P (i.e., $P|\psi\rangle = |\psi\rangle$).

d) Find a property $P = A_1 \otimes B_1 + A_2 \otimes B_2$, where the A_j and B_k are one dimensional projectors and P is a two-dimensional projector, such that $|\psi\rangle$ has property P. What is the physical meaning of P, in words? It is actually very difficult to give a precise definition, but try and express the general idea.

Sample answer to Exercise 1 by Willy Smart

I glanced at the linear algebra stuff in QCQI Sec. 2.1 and Ch. 3 of CQT, and it all looked familiar from my linear algebra course, except that they are using complex numbers rather than real numbers, and this isn't very important, is it? [Comment by instructor: yes it is — you had better take another look!]

I also read Secs. 4.1 to 4.4 of QCQI, though for 4.3 and 4.4 it was more glancing through than really reading them. I did exercises 4.1, 4.2, and 4.7, but got stuck on exercise 4.5: what does $(\hat{n}\vec{\sigma})^2$ mean? In Sec. 4.4 I did not understand what they mean by the principles of deferred measurement and implicit measurements. The statements didn't seem to make much sense, and I have no idea how to do exercise 4.35.

Chapter 4 in CQT seemed straightforward until I got to Sec. 4.6, which was quite confusing and left me feeling a bit uncertain: is that the uncertainty principle? Guess there's got to be something weird about the quantum world! Hey, prof, do you believe in Schrödinger's cat?

Complaints: Instructor tells too many jokes. Also, the pace of the course has been a bit *fast*. Except, of course, for the stuff I'd already seen before, where the instructor has been wasting everybody's time with all those details.