Chapter 13

Quantum Interference

13.1 Two-Slit and Mach-Zehnder Interferometers

Interference effects involving quantum particles reflect both the wave-like and particle-like properties of quantum entities. One of the best-known examples is the interference pattern produced by a double slit. Quantum particles—photons or neutrons or electrons—are sent one at a time through the slit system shown in Fig. 13.1, and later arrive at a series of detectors located in the diffraction zone far from the slits. The detectors are triggered at random, with each particle triggering just one detector. After enough particles have been detected, an interference pattern can be discerned in the relative counting rates of the different detectors, indicated by the length of the horizontal bars in the figure. Lots of particles arrive at some detectors, very few particles at others.

Figure 13.1: Interference pattern for a wave arriving from the left and passing through the two slits. Each circle on the right side represents a detector, and the black bar to its right indicates the relative counting rate.

The relative number of particles arriving at each detector depends on the *difference* of the distances between the detector and the two slits, in units of the particle's de Broglie wavelength. Furthermore, this interference pattern persists even at very low intensities, say one particle per second passing through the slit system. Hence it seems very unlikely that it arises from a sort of cooperative phenomenon in which a particle going through one slit compares notes with a particle going through the other slit. Instead, each particle must somehow pass through both slits, for how else can one understand the interference effect?

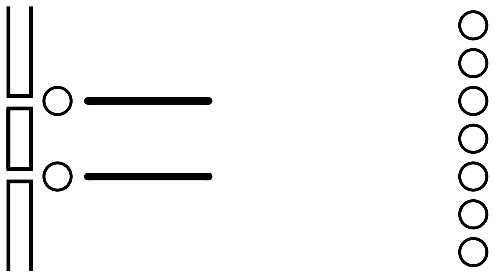

Figure 13.2: Detectors directly behind the two slits. The black bars are again proportional to the counting rates.

However, if detectors are placed directly behind the two slits, Fig. 13.2, then either one or the other detector detects a particle, and it is never the case that both detectors simultaneously detect a particle. Furthermore, the total counting rate for the arrangement in Fig. 13.2 is the same as that in Fig. 13.1, suggesting that if a particle had not been detected just behind one of the slits, it would have continued on into the diffraction zone and arrived at one of the detectors located there. Thus it seems plausible that the particles which do arrive in the diffraction zone in Fig. 13.1 have earlier passed through one or the other of the two slits, and not both. But this is difficult to reconcile with the interference effect seen in the diffraction zone, which seems to require that each particle pass through both slits. Could a particle passing through one slit somehow sense the presence of the other slit, and take this into account when it arrives in the diffraction zone?

Figure 13.3: A light source L between the slits washes out the electron interference pattern.

In Feynman's discussion of two-slit interference (see bibliography), he considers what happens if there is a non-destructive measurement of which slit the particle passes through, a measurement that allows the particle to continue on its way and later be detected in the diffraction zone. His quantum particles are electrons, and he places a light source just behind the slits, Fig. 13.3. By scattering a photon off the electron one can "see" which slit it has just passed through. Illuminating the slits in this way washes out the interference effect, and the intensities in the diffraction zone can be explained as sums of intensities due to electrons coming through each of the two slits.

Feynman then imagines reducing the intensity of the light source to such a degree that sometimes an electron scatters a photon, revealing which slit it passed through, and sometimes it does not. Data for electrons arriving in the diffraction zone are then segregated into two sets: one set for "visible" electrons which earlier scattered a photon, and the other for electrons which were "invisible" as they passed through the slit system. When the set of data for the "visible" electrons is examined it shows no interference effects, whereas that for the "invisible" electrons indicates that they arrive in the diffraction zone with the same interference pattern as when there is no source of light behind the slits. Can the behavior of an electron really depend upon whether or not it has been seen?

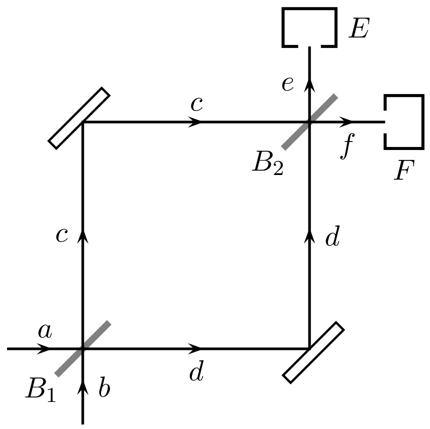

Figure 13.4: Mach-Zehnder interferometer with detectors. The beam splitters are labeled B_1 and B_2.

In this chapter we explore these paradoxes using a toy Mach-Zehnder interferometer, which exhibits the same sorts of paradoxes as a double slit, but is easier to discuss. A Mach-Zehnder interferometer, Fig. 13.4, consists of a beam splitter followed by two mirrors which bring the split beams back together again, and a second beam splitter placed where the reflected beams intersect. Detectors can be placed on the output channels. We assume that light from a monochromatic source enters the first beam splitter through the a channel. The intensity of light emerging in the two output channels e and f depends on the *difference* in path length, measured in units of the wavelength of the light, in the c and d arms of the interferometer. (The classical wave theory of light suffices for calculating these intensities; one does not need quantum theory.) We shall assume that this difference has been adjusted so that after the second beam splitter all the light which enters through the a channel emerges in the f channel and none in the e channel. Rather than changing the physical path lengths, it is possible to alter the final intensities by inserting *phase shifters* in one or both arms of the interferometer. (A phase shifter is a piece of dielectric material which, when placed in the light beam, alters the optical path length (number of wavelengths) between the two beam splitters.)

An interferometer for neutrons which is analogous to a Mach-Zehnder interferometer for photons can be constructed using a single crystal of silicon. For our purposes the difference between these two types of interferometer is not important, since neutrons are quantum particles that behave like waves, and photons are light waves that behave like particles. Thus while we shall continue to think of photons going through a Mach-Zehnder interferometer, the toy model introduced in Sec. 13.2 below could equally well describe the interference of neutrons.

The analogy between a Mach-Zehnder interferometer and double-slit interference is the following. Each photon on its way through the interferometer must pass through the c arm or the d arm in much the same way that a particle (photon or something else) must pass through one of the two slits on its way to a detector in the diffraction zone. The first beam splitter provides a source of coherent light (that is, the relative phase is well defined) for the two arms of the interferometer, just as one needs a coherent source of particles illuminating the two slits. (This coherent source can be a single slit a long distance to the left of the double slit.) The second beam splitter in the interferometer combines beams from the separate arms and makes them interfere in a way which is analogous to the interference of the beams emerging from the two slits when they reach the diffraction zone.

13.2 Toy Mach-Zehnder Interferometer

We shall set up a stochastic or probabilistic model of a toy Mach-Zehnder interferometer, Fig. 13.5, and discuss what happens when a *single* particle or photon passes through the instrument. The model will supply us with probabilities for different possible histories of this single particle. If one imagines, as in a real experiment, lots of particles going through the apparatus, one after another, then each particle represents an "independent trial" in the sense of probability theory. That is, each particle will follow (or undergo) a particular history chosen randomly from the collection of all possible histories. If a large number of particles are used, then the number which follow some given history will be proportional to the probability, computed by the laws of quantum theory, that a single particle will follow that history.

The toy Mach-Zehnder interferometer consists of two toy beam splitters, of the type shown in Fig. 12.1 in Sec. 12.1, in series. The arms and the entrance and output channels are labeled in a way which corresponds to Fig. 13.4. The unitary time transformation for the toy model is $T = S_i$, where the operator S_i is defined by

$$S_i|mz\rangle = |(m+1)z\rangle \tag{13.1}$$

for m an integer, and $z = a, b, c, d, e$ or f, with the exceptions

$$\begin{aligned} S_i|0a\rangle &= \big(+|1c\rangle + |1d\rangle\big)/\sqrt{2}, \quad S_i|0b\rangle = \big(-|1c\rangle + |1d\rangle\big)/\sqrt{2}, \\ S_i|3c\rangle &= \big(+|4e\rangle + |4f\rangle\big)/\sqrt{2}, \quad S_i|3d\rangle = \big(-|4e\rangle + |4f\rangle\big)/\sqrt{2}. \end{aligned} \tag{13.2}$$

(See the comment following (12.2) on the choice of phases.) In addition, the usual provision must be made for periodic boundary conditions, but (as usual) these will not play any role in the discussion which follows; see the remarks in Sec. 12.1. The transformation S_i is unitary because it maps an

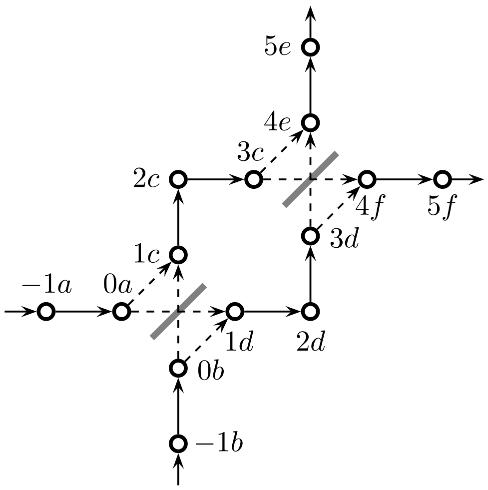

Figure 13.5: Toy Mach-Zehnder interferomter constructed from two beam splitters of the sort shown in Fig. 12.1.

orthonormal basis, the collection of states $\{|mz\rangle\}$, onto an orthonormal basis of the Hilbert space. A particle (photon) which enters the a channel undergoes a unitary time evolution of the form

$$|0a\rangle \mapsto |1\bar{a}\rangle \mapsto |2\bar{a}\rangle \mapsto |3\bar{a}\rangle \mapsto |4f\rangle \mapsto |5f\rangle \cdots, \tag{13.3}$$

where, as in (12.6),

$$|m\bar{a}\rangle = \big(+|mc\rangle + |md\rangle\big)/\sqrt{2}, \quad |m\bar{b}\rangle = \big(-|mc\rangle + |md\rangle\big)/\sqrt{2} \tag{13.4}$$

are superpositions of states of the particle in the c and d arms of the interferometer, with phases chosen to correspond to unitary evolution under S_i starting with $|0a\rangle$, and $|0b\rangle$, respectively.

The probability that the particle emerges in the e or in the f channel is influenced by what happens in *both* arms of the interferometer, as can be seen in the following way. Let us introduce *toy phase shifters* in the c and d arms by using in place of S_i a unitary time transformation S_i' identical to S_i, (13.1) and (13.2), except that

$$S_i'\,|2c\rangle = \exp(i\phi_c)\,|3c\rangle, \quad S_i'\,|2d\rangle = \exp(i\phi_d)\,|3d\rangle, \tag{13.5}$$

where ϕ_c and ϕ_d are phase shifts. Obviously S_i' is unitary, and it is the same as S_i when ϕ_c and ϕ_d are zero. If we use S_i' in place of S_i, the unitary time evolution in (13.3) becomes:

$$\begin{aligned} |0a\rangle &\mapsto |1\bar{a}\rangle \mapsto |2\bar{a}\rangle = \big(|2c\rangle + |2d\rangle\big)/\sqrt{2} \mapsto \big(e^{i\phi_c}|3c\rangle + e^{i\phi_d}|3d\rangle\big)/\sqrt{2} \\ &\mapsto \tfrac{1}{2}\Big[\big(e^{i\phi_c} - e^{i\phi_d}\big)|4e\rangle + \big(e^{i\phi_c} + e^{i\phi_d}\big)|4f\rangle\Big] \mapsto \cdots, \end{aligned} \tag{13.6}$$

where the result at $t = 5$ is obtained by replacing $|4e\rangle$ by $|5e\rangle$, and $|4f\rangle$ by $|5f\rangle$.

Consider a consistent family of histories based upon an initial state $|0a\rangle$ at $t = 0$ and a decomposition of the identity corresponding to the orthonormal basis $\{|mz\rangle\}$ at a second time $t = 4$. There are two histories with positive weight,

$$Y = [0a]_0 \odot [4e]_4, \quad Y' = [0a]_0 \odot [4f]_4, \tag{13.7}$$

where, as usual, subscripts indicate the time. The probabilities can be read off from the $t = 4$ term in (13.6), treating it as a pre-probability, by taking the absolute squares of the coefficients of $|4e\rangle$ and $|4f\rangle$:

$$\begin{aligned} \Pr([4e]_4) = \Pr(Y) &= |e^{i\phi_c} - e^{i\phi_d}|^2/4 = [\sin(\Delta\phi/2)]^2, \\ \Pr([4f]_4) = \Pr(Y') &= |e^{i\phi_c} + e^{i\phi_d}|^2/4 = [\cos(\Delta\phi/2)]^2, \end{aligned} \tag{13.8}$$

where

$$\Delta\phi = \phi_c - \phi_d \tag{13.9}$$

is the difference between the two phase shifts. Since these probabilities depend upon $\Delta\phi$, and thus upon what is happening in *both* arms of the interferometer, the quantum particle must in some sense be delocalized as it passes through the interferometer, rather than localized in arm c or in arm d. On the other hand, it is a mistake to think of it as simultaneously present in both arms in the sense that "it is in c *and at the same time* it is in d." See the remarks in Sec. 4.5: a quantum particle cannot be in two places at the same time.

Similarly, if we want to understand double-slit interference using this analogy, we would like to say that the particle "goes through both slits," without meaning that it is present in the upper slit at the same time as it is present in the lower slit, or that it went through one slit or the other and we do not know which. See the discussion of the localization of quantum particles in Secs. 2.3 and 4.5. Speaking of the particle as "passing through the slit system" conveys roughly the right meaning. In the double slit experiment, one could introduce phase shifters behind each slit, and thereby shift the positions of the peaks and valleys of the interference pattern in the diffraction zone. Again, it is the *difference* of the phase shifts which is important, and this shows that one somehow has to think of the quantum particle as a coherent entity as it passes through the slit system.

Very similar results are obtained if instead of $|0a\rangle$ one uses a wave packet

$$|\psi_0\rangle = c\,|{-2a}\rangle + c'|{-1a}\rangle + c''|0a\rangle, \tag{13.10}$$

in the a channel as the initial state at $t = 0$, where c, c', and c'' are numerical coefficients. For such an initial state it is convenient to use histories

$$X = [\psi_0] \odot E_t, \quad X' = [\psi_0] \odot F_t \tag{13.11}$$

rather than Y and Y' in (13.7), where

$$E = \sum_m [me], \quad F = \sum_m [mf] \tag{13.12}$$

are projectors for the particle to be someplace in the e and f channels, respectively, and E_t means the particle is in the e channel at time t; see the analogous (12.16). As long as $t \geq 6$, so that the entire wave packet corresponding to $|\psi_0\rangle$ has a chance to emerge from the interferometer, one finds that the corresponding probabilities are

$$\begin{aligned} \Pr(E_t) = \Pr(X) &= [\sin(\Delta\phi/2)]^2, \\ \Pr(F_t) = \Pr(X') &= [\cos(\Delta\phi/2)]^2, \end{aligned} \tag{13.13}$$

precisely the same as in (13.8). Since the philosophy behind toy models is simplicity and physical insight, not generality, we shall use only the simple initial state $|0a\rangle$ in what follows, even though a good part of the discussion would hold (with some fairly obvious modifications) for a more general initial state representing a wave packet entering the interferometer in the a channel.

What can we say about the particle while it is *inside* the interferometer, during the time interval for which the histories in (13.7) provide no information? There are various ways of refining these histories by inserting additional events at times between $t = 0$ and 4. For example, one can employ unitary extensions, Sec. 11.7, of Y and Y' by using the unitary time development of the initial $|0a\rangle$ at intermediate times to obtain two histories

$$\begin{aligned} Y^e &= [0a] \odot [1\bar{a}] \odot [2\bar{a}] \odot [3\bar{q}] \odot [4e], \\ Y^f &= [0a] \odot [1\bar{a}] \odot [2\bar{a}] \odot [3\bar{q}] \odot [4f], \end{aligned} \tag{13.14}$$

defined at $t = 0, 1, 2, 3, 4$, which form the support of a consistent family with initial state $[0a]$. The projector $[3\bar{q}]$ is onto the state

$$|3\bar{q}\rangle := \big(e^{i\phi_c}|3c\rangle + e^{i\phi_d}|3d\rangle\big)/\sqrt{2}. \tag{13.15}$$

The histories in (13.14) are identical up to $t = 3$, and then split. One can place the split earlier, between $t = 2$ and $t = 3$, by mapping $[4e]$ and $[4f]$ unitarily backwards in time to $t = 3$:

$$\begin{aligned} \bar{Y}^e &= [0a] \odot [1\bar{a}] \odot [2\bar{a}] \odot [3\bar{b}] \odot [4e], \\ \bar{Y}^f &= [0a] \odot [1\bar{a}] \odot [2\bar{a}] \odot [3\bar{a}] \odot [4f]. \end{aligned} \tag{13.16}$$

Note that Y, Y^e, and $\bar{Y}^e$ all have exactly the same chain operator, for reasons discussed in Sec. 11.7, and the same is true of Y', Y^f, and $\bar{Y}^f$. The consistency of the family (13.7) is automatic, as only two times are involved, Sec. 11.3. As a consequence the unitary extensions (13.14) and (13.16) of that family are supports of consistent families; see Sec. 11.7.

The families in (13.14) and (13.16) can be used to discuss some aspects of the particle's behavior while inside the interferometer, but cannot tell us whether it was in the c or in the d arm, because the projectors C and D, (12.9), do not commute with projectors onto superposition states, such as $[1\bar{a}]$, $[3\bar{q}]$, or $[3\bar{b}]$. Instead, we must look for alternative families in which events of the form $[mc]$ or $[md]$ appear at intermediate times. It will simplify the discussion if we assume that $\phi_c = 0 = \phi_d$, i.e., use S_i for time development rather than the more general S_i'.

One consistent family of this type has for its support the two elementary histories

$$\begin{aligned} Y^c &= [0a] \odot [1c] \odot [2c] \odot [3c] \odot [4\bar{c}] \odot [5\bar{c}] \odot \cdots [\tau\bar{c}], \\ Y^d &= [0a] \odot [1d] \odot [2d] \odot [3d] \odot [4\bar{d}] \odot [5\bar{d}] \odot \cdots [\tau\bar{d}], \end{aligned} \tag{13.17}$$

where

$$|m\bar{c}\rangle = \big(+|me\rangle + |mf\rangle\big)/\sqrt{2}, \quad |m\bar{d}\rangle = \big(-|me\rangle + |mf\rangle\big)/\sqrt{2} \tag{13.18}$$

for $m \geq 4$ correspond to unitary time evolution starting with $|3c\rangle$ and $|3d\rangle$, respectively. The final time τ can be as large as one wants, consistent with the particle not having passed out of the e or f channels due to the periodic boundary condition. The histories in (13.17) are unitary extensions of $[0a] \odot [1c]$ and $[0a] \odot [1d]$, and consistency follows from the general arguments given in Sec. 11.7.

Note that if we use Y^c and Y^d, we cannot say whether the particle emerges in the e or f channel of the second beam splitter, whereas if we use Y^e and Y^f in (13.14), with $\phi_c = 0 = \phi_d$, we can say that the particle leaves this beam splitter in a definite channel, but we cannot discuss the channel in which it arrives at the beam splitter.

In order to describe the particle as being in a definite arm of the interferometer *and* emerging in a definite channel from the second beam splitter, one might try a family which includes

$$\begin{aligned} Y^{ce} &= [0a] \odot [1c] \odot [2c] \odot [3c] \odot [4e] \odot [5e] \odot \cdots [\tau e], \\ Y^{cf} &= [0a] \odot [1c] \odot [2c] \odot [3c] \odot [4f] \odot [5f] \odot \cdots [\tau f], \\ Y^{de} &= [0a] \odot [1d] \odot [2d] \odot [3d] \odot [4e] \odot [5e] \odot \cdots [\tau e], \\ Y^{df} &= [0a] \odot [1d] \odot [2d] \odot [3d] \odot [4f] \odot [5f] \odot \cdots [\tau f], \end{aligned} \tag{13.19}$$

continuing till some final time τ. Alas, this will not work. The family is inconsistent, because

$$\langle K(Y^{ce}), K(Y^{de})\rangle \neq 0, \quad \langle K(Y^{cf}), K(Y^{df})\rangle \neq 0, \tag{13.20}$$

as is easily shown using the corresponding chain kets (Sec. 11.6). In fact, each of the histories in (13.19) is *intrinsically inconsistent* in the sense that there is no way of making it part of some consistent family. See the discussion of intrinsic inconsistency in Sec. 11.8; the strategy used there for histories involving three times is easily extended to cover the somewhat more complicated situation represented in (13.19).

The analog of (13.14) for two-slit interference is a consistent family $\mathcal{F}$ in which the particle passes through the slit system in a delocalized state, but arrives at a definite location in the diffraction zone. It is $\mathcal{F}$ which lies behind conventional discussions of two-slit interference, which emphasize (correctly) that in such circumstances it is meaningless to discuss which slit the particle passed through. However, there is also another consistent family $\mathcal{G}$, the analog of (13.17), in which the particle passes through one or the other of the two slits, and is described in the diffraction zone by one of two delocalized wave packets, the counterparts of the $\bar{c}$ and $\bar{d}$ superpositions defined in (13.18). Although these wave packets overlap in space, they are orthogonal to each other and thus represent distinct quantum states. The families $\mathcal{F}$ and $\mathcal{G}$ are incompatible, and hence the descriptions they provide cannot be combined. Attempting to do so by assuming that the particle goes through a definite slit *and* arrives at a definite location in the diffraction zone gives rise to inconsistencies analogous to those noted in connection with (13.19).

From the perspective of fundamental quantum theory there is no reason to prefer one of these two families to the other. Each has its use for addressing certain types of physical question. If one wants to know the location of the particle when it reaches the diffraction zone, $\mathcal{F}$ must be used in preference to $\mathcal{G}$, because it is only in $\mathcal{F}$ that this location makes sense. On the other hand, if one wants to know which slit the particle passed through, $\mathcal{G}$ must be employed, for in $\mathcal{F}$ the concept of passing through a particular slit makes no sense. Experiments can be carried out to check the predictions of either family, and the Mach-Zehnder analogs of these two kinds of experiments are discussed in the next two sections.

13.3 Detector in Output of Interferometer

Let us add to the e output channel of our toy Mach Zehnder interferometer a simple two-state detector of the type introduced in Sec. 7.4 and used in Sec. 12.2, see Fig. 12.2. The detector states are $|0\hat{e}\rangle$, "ready", and $|1\hat{e}\rangle$, "triggered", and the unitary time development operator is

$$T = S_i' R_e, \tag{13.21}$$

where R_e is the identity on the Hilbert space $\mathcal{M} \otimes \mathcal{E}$ of particle-plus-detector, except for

$$R_e|4e, n\hat{e}\rangle = |4e, (1-n)\hat{e}\rangle, \tag{13.22}$$

with $n = 0$ or 1, which is the analog of (12.27). Thus, in particular,

$$T|4e, 0\hat{e}\rangle = |5e, 1\hat{e}\rangle, \quad T|4f, 0\hat{e}\rangle = |5f, 0\hat{e}\rangle, \tag{13.23}$$

so the detector is triggered by the particle emerging in the e channel as it hops from $4e$ to $5e$, but is not triggered if the particle emerges in the f channel. We could add a second detector for the f channel, but that is not necessary: if the e channel detector remains in its ready state after a certain time, that will tell us that the particle emerged in the f channel. See the discussion in Sec. 12.2.

Assume an initial state

$$|\Psi_0\rangle = |0a, 0\hat{e}\rangle, \tag{13.24}$$

and consider histories that are the obvious counterparts of those in (13.14),

$$\begin{aligned} Z^e &= Z^i \odot [4e, 0\hat{e}] \odot [5e, 1\hat{e}] \odot [6e, 1\hat{e}] \odot \cdots [\tau e, 1\hat{e}], \\ Z^f &= Z^i \odot [4f, 0\hat{e}] \odot [5f, 0\hat{e}] \odot [6f, 0\hat{e}] \odot \cdots [\tau f, 0\hat{e}], \end{aligned} \tag{13.25}$$

but which continue on to some final time τ. The initial unitary portion

$$Z^i = [\Psi_0] \odot [1\bar{a}, 0\hat{e}] \odot [2\bar{a}, 0\hat{e}] \odot [3\bar{q}, 0\hat{e}] \tag{13.26}$$

is the same for both Z^e and Z^f. The histories in (13.25) are the support of a consistent family with initial state $|\Psi_0\rangle$, and they contain no surprises. If the particle passes through the interferometer in a coherent superposition and emerges in channel e, it triggers the detector and keeps going. If it emerges in f it does not trigger the detector, and continues to move out that channel. The probability that the detector will be in its triggered state at $t = 5$ or later is $\sin^2(\Delta\phi/2)$, the same as the probability calculated earlier, (13.8), that the particle will emerge in the e channel when no detector is present.

As a second example, suppose that $\phi_c = 0 = \phi_d$, and consider the consistent family whose support consists of the two histories

$$\begin{aligned} Z^c &= [\Psi_0] \odot [1c] \odot [2c] \odot [3c] \odot [4\bar{c}, 0\hat{e}] \odot [5r] \odot [6r] \odot \cdots [\tau r], \\ Z^d &= [\Psi_0] \odot [1d] \odot [2d] \odot [3d] \odot [4\bar{d}, 0\hat{e}] \odot [5s] \odot [6s] \odot \cdots [\tau s], \end{aligned} \tag{13.27}$$

where the detector state $[0\hat{e}]$ has been omitted for times earlier than $t = 4$ (it could be included at all these times in both histories), and

$$|mr\rangle = \frac{|me, 1\hat{e}\rangle + |mf, 0\hat{e}\rangle}{\sqrt{2}}, \quad |ms\rangle = \frac{-|me, 1\hat{e}\rangle + |mf, 0\hat{e}\rangle}{\sqrt{2}} \tag{13.28}$$

are superpositions of states in which the detector has and has not been triggered, so they are toy MQS (macroscopic quantum superposition) states, as in (12.29). The histories in (13.27) are obvious counterparts of those in (13.17), and they are unitary extensions (Sec. 11.7) to later times of $[\Psi_0] \odot [1c, 0\hat{e}]$, and $[\Psi_0] \odot [1d, 0\hat{e}]$.

The toy MQS states at time $t \geq 5$ in (13.27) are hard to interpret, and their grown-up counterparts for a real Mach-Zehnder or neutron interferometer are impossible to observe in the laboratory. Can we get around this manifestation of Schrödinger's cat (Sec. 9.6) by the same method we used in Sec. 12.2: using histories in which the detector is in its pointer basis (see the definition at the end of Sec. 9.5) rather than in some MQS state? The obvious choice would be something like

$$\begin{aligned}
Z^{ce} &= [\Psi_0] \odot [1c] \odot [2c] \odot [3c] \odot [4e, 0\hat{e}] \odot [5e, 1\hat{e}] \odot \cdots, \\
Z^{cf} &= [\Psi_0] \odot [1c] \odot [2c] \odot [3c] \odot [4f, 0\hat{e}] \odot [5f, 0\hat{e}] \odot \cdots, \\
Z^{de} &= [\Psi_0] \odot [1d] \odot [2d] \odot [3d] \odot [4e, 0\hat{e}] \odot [5e, 1\hat{e}] \odot \cdots, \\
Z^{df} &= [\Psi_0] \odot [1d] \odot [2d] \odot [3d] \odot [4f, 0\hat{e}] \odot [5f, 0\hat{e}] \odot \cdots,
\end{aligned} \tag{13.29}$$

where, once again, we have omitted the detector state $[0\hat{e}]$ at times earlier than $t = 4$. However, this family is inconsistent: (13.20) holds with Y replaced by Z, and one can even show that the individual histories in (13.29), like those in (13.19), are intrinsically inconsistent. Indeed, the history

$$[\Psi_0]_0 \odot C_t \odot [1\hat{e}]_{t'}, \tag{13.30}$$

in which the initial state is followed by a particle in the c arm at some time in the interval $1 \leq t \leq 3$, and then the detector in its triggered state at a later time $t' \geq 5$, is intrinsically inconsistent, and the same is true if C_t is replaced by D_t, or $[1\hat{e}]_{t'}$ by $[0\hat{e}]_{t'}$. (For the meaning of C_t or D_t, see the discussion following (12.15).)

A similar analysis can be applied to the analogous situation of two-slit interference in which a detector is located at some point in the diffraction zone. By using a family in which the particle passes through the slit system in a delocalized state corresponding to unitary time evolution, the analog of (13.25), one can show that the probability of detection is the same as the probability of the particle arriving at the corresponding region in space in the absence of a detector. There is also a family, the analog of (13.27), in which the particle passes through a definite slit, and later on the detector is described by an MQS state, the counterpart of one of the states defined in (13.28). There is no way of "collapsing" these MQS states into pointer states of the detector—this is the lesson to be drawn from the inconsistent family (13.29)—as long as one insists upon assigning a definite slit to the particle.

This example shows that it is possible to construct families of histories using events at earlier times which are "normal" (non-MQS), but which have the consequence that at later times one is "forced" to employ MQS states. If one does not want to use MQS states at a later time, it is necessary to change the events in the histories at earlier times, or alter the initial states. Note that consistency depends upon *all* the events which occur in a history, because the chain operator depends upon all the events, so one cannot say that inconsistency is "caused" by a particular event in the history, unless one has decided that other events shall, by definition, not share in the blame.

13.4 Detector in Internal Arm of Interferometer

Let us see what happens if a detector is placed in the c arm inside the toy interferometer. (A detector could also be placed in the d arm, but this would not lead to anything new, since if the particle is not detected in the c arm one can conclude that it passed through the d arm.) The detector states are $|0\hat{c}\rangle$ "ready" and $|1\hat{c}\rangle$ "triggered". The unitary time operator is

$$T = S_i' R_c, \tag{13.31}$$

where S_i' is defined in (13.5), and R_c is the identity on the space $\mathcal{M} \otimes \mathcal{C}$ of particle and detector, except for

$$R_c|2c, n\hat{c}\rangle = |2c, (1-n)\hat{c}\rangle. \tag{13.32}$$

In particular,

$$T|2c, 0\hat{c}\rangle = e^{i\phi_c}|3c, 1\hat{c}\rangle, \quad T|2d, 0\hat{c}\rangle = e^{i\phi_d}|3d, 0\hat{c}\rangle, \tag{13.33}$$

so the detector is triggered as the particle hops from $2c$ to $3c$ when passing through the c arm, but is not triggered if the particle passes through the d arm.

Consider the unitary time development,

$$|\Phi_t\rangle = T^t|\Phi_0\rangle, \quad |\Phi_0\rangle = |0a, 0\hat{c}\rangle, \tag{13.34}$$

of an initial state in which the particle is in the a channel, and the c channel detector is in its ready state. At $t = 4$ we have:

$$|\Phi_4\rangle = \tfrac{1}{2}\big[e^{i\phi_c}|4e, 1\hat{c}\rangle - e^{i\phi_d}|4e, 0\hat{c}\rangle + e^{i\phi_c}|4f, 1\hat{c}\rangle + e^{i\phi_d}|4f, 0\hat{c}\rangle\big], \tag{13.35}$$

where all four states in the sum on the right side are mutually orthogonal.

One can use (13.35) as a pre-probability to compute the probabilities of two-time histories beginning with the initial state $|\Phi_0\rangle$ at $t = 0$, and with the particle in either the e or in the f channel at $t = 4$. Thus consider a family in which the four histories with non-zero weight are of the form $\Phi_0 \odot [\phi_j]$, where $|\phi_j\rangle$ is one of the four kets on the right side of (13.35). Each will occur with probability 1/4, and thus

$$\Pr([4e]_4) = 1/4 + 1/4 = 1/2 = \Pr([4f]_4). \tag{13.36}$$

Upon comparing these with (13.8) when no detector is present, one sees that inserting a detector in one arm of the interferometer has a drastic effect: there is no longer any dependence of these probabilities upon the phase difference $\Delta\phi$. Thus a measurement of which arm the particle passes through wipes out all the interference effects which would otherwise be apparent in the output intensities following the second beam splitter. Note the analogy with Feynman's discussion of the double slit: determining which slit the electron goes through, by scattering light off of it, destroys the interference pattern in the diffraction zone.

Now let us consider various possible histories describing what the particle does while it is inside the interferometer, assuming $\phi_c = 0 = \phi_d$ in order to simplify the discussion. Straightforward unitary time evolution will result in a family in which every $[\Phi_t]$ for $t \geq 3$ is a toy MQS state involving both $|0\hat{c}\rangle$ and the triggered state $|1\hat{c}\rangle$ of the detector. In order to obtain a consistent family without MQS states, we can let unitary time development continue up until the measurement occurs, and then have a split (or collapse) to produce the analog of (12.33)) in the previous chapter: a family whose support consists of the two histories

$$\begin{aligned} V^c &= [0a, 0\hat{c}] \odot [1\bar{a}, 0\hat{c}] \odot [2\bar{a}, 0\hat{c}] \odot [3c, 1\hat{c}] \odot [4\bar{c}, 1\hat{c}] \odot \cdots , \\ V^d &= [0a, 0\hat{c}] \odot [1\bar{a}, 0\hat{c}] \odot [2\bar{a}, 0\hat{c}] \odot [3d, 0\hat{c}] \odot [4\bar{d}, 0\hat{c}] \odot \cdots , \end{aligned} \tag{13.37}$$

with states $|m\bar{c}\rangle$ and $|m\bar{d}\rangle$ defined in (13.18). One can equally well put the split at an earlier time, by using histories

$$\begin{aligned} \bar{Z}^c &= [0a, 0\hat{c}] \odot [1c, 0\hat{c}] \odot [2c, 0\hat{c}] \odot [3c, 1\hat{c}] \odot [4\bar{c}, 1\hat{c}] \odot \cdots , \\ \bar{Z}^d &= [0a, 0\hat{c}] \odot [1d, 0\hat{c}] \odot [2d, 0\hat{c}] \odot [3d, 0\hat{c}] \odot [4\bar{d}, 0\hat{c}] \odot \cdots , \end{aligned} \tag{13.38}$$

which resemble those in (13.17) in that the particle is in the c or in the d arm from the moment it leaves the first beam splitter.

One can also introduce a second split at the second beam splitter, to produce a family with support

$$\begin{aligned} \bar{Z}^{ce} &= [0a, 0\hat{c}] \odot [1c, 0\hat{c}] \odot [2c, 0\hat{c}] \odot [3c, 1\hat{c}] \odot [4e, 1\hat{c}] \odot [5e, 1\hat{c}] \cdots , \\ \bar{Z}^{cf} &= [0a, 0\hat{c}] \odot [1c, 0\hat{c}] \odot [2c, 0\hat{c}] \odot [3c, 1\hat{c}] \odot [4f, 1\hat{c}] \odot [5f, 1\hat{c}] \cdots , \\ \bar{Z}^{de} &= [0a, 0\hat{c}] \odot [1d, 0\hat{c}] \odot [2d, 0\hat{c}] \odot [3d, 0\hat{c}] \odot [4e, 0\hat{c}] \odot [5e, 0\hat{c}] \cdots , \\ \bar{Z}^{df} &= [0a, 0\hat{c}] \odot [1d, 0\hat{c}] \odot [2d, 0\hat{c}] \odot [3d, 0\hat{c}] \odot [4f, 0\hat{c}] \odot [5f, 0\hat{c}] \cdots . \end{aligned} \tag{13.39}$$

This family is consistent, in contrast to (13.19), because the projectors of the different histories at some final time τ are mutually orthogonal: the orthogonal final states of the detector prevent the inconsistency which would arise, as in (13.20), if one only had particle states. In addition, one could place another detector in one of the output channels. However, when used with a family analogous to (13.39) this detector would simply confirm the arrival of the particle in the corresponding channel with the same probability as if the detector had been absent, so one would learn nothing new.

Inserting a detector into the c arm of the interferometer provides an instance of what is often called *decoherence*. The states $|m\bar{a}\rangle$ and $|m\bar{b}\rangle$ defined in (13.4) are *coherent* superpositions of the states $|mc\rangle$ and $|md\rangle$ in which the particle is localized in one or the other arm of the interferometer, and the relative phases in the superposition are of physical significance, since in the absence of a detector one of these superpositions will result in the particle emerging in the f channel, and the other in its emerging in e. However, when something like a cosmic ray interacts with the particle in a sufficiently different way in the c and the d arm, it destroys the coherence (the influence of the relative phase), and thus produces decoherence.

The scattering of light in Feynman's version of the double slit experiment is an example of decoherence in this sense, and it results in interference effects being washed out. However, decoherence is usually not an "all or nothing" affair. The weakly-coupled detectors discussed in Sec. 13.5 below provide an example of *partial decoherence*. As well as washing out interference effects, decoherence can expand the range of possibilities for constructing consistent families. Thus the family based on (13.19) in which the particle is in a definite arm inside the interferometer and emerges from the interferometer in a definite channel is inconsistent, whereas its counterpart in (13.39), with decoherence taking place inside the interferometer, is consistent. Some additional discussion of decoherence will be found in Ch. 26.

13.5 Weak Detectors in Internal Arms

As noted in Sec. 13.1, Feynman in his discussion of double-slit interference tells us that as the intensity of the light behind the double slits is reduced, one will find that those electrons which do not scatter a photon will, when they arrive in the diffraction zone, exhibit the same interference pattern as when the light is off. Let us try and understand this effect by placing *weakly-coupled* or *weak* detectors in the c and d arms of the toy Mach-Zehnder interferometer.

A simple toy weak detector has two orthogonal states, $|0\hat{c}\rangle$ "ready" and $|1\hat{c}\rangle$ "triggered", and the weak coupling is arranged by replacing the unitary transformation R_c in (13.32) with R'_c, which is the identity except for

$$\begin{aligned} R'_c|2c,0\hat{c}\rangle &= \alpha|2c,0\hat{c}\rangle + \beta|2c,1\hat{c}\rangle, \\ R'_c|2c,1\hat{c}\rangle &= \gamma|2c,0\hat{c}\rangle + \delta|2c,1\hat{c}\rangle, \end{aligned} \tag{13.40}$$

where α, β, γ, and δ are (in general complex) numbers forming a unitary 2×2 matrix

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}. \tag{13.41}$$

The "strongly-coupled" or "strong" detector used previously is a special case in which $\beta = 1 = \gamma$, $\alpha = \delta = 0$. Making $|\beta|$ small results in a weak coupling, since the probability that the detector will be triggered by the presence of a particle at site $2c$ is $|\beta|^2$. (One can also modify the time-elapse detector of Sec. 12.3 to make it a weakly-coupled detector, by modifying (12.40) in a manner analogous to (13.40), but we will not need it for the present discussion.) It is convenient for purposes of exposition to assume a symmetrical arrangement in which there is a second detector, with ready and triggered states $|0\hat{d}\rangle$ and $|1\hat{d}\rangle$, in the d arm of the interferometer, with its coupling to the particle governed by a unitary transformation R'_d equal to the identity except for

$$\begin{aligned} R'_d|2d,0\hat{d}\rangle &= \alpha|2d,0\hat{d}\rangle + \beta|2d,1\hat{d}\rangle, \\ R'_d|2d,1\hat{d}\rangle &= \gamma|2d,0\hat{d}\rangle + \delta|2d,1\hat{d}\rangle, \end{aligned} \tag{13.42}$$

where the numerical coefficients α, β, γ, and δ are the same as in (13.40).

The overall unitary time development of the entire system $\mathcal{M}\otimes\mathcal{C}\otimes\mathcal{D}$ consisting of the particle and the two detectors is determined by the operator

$$T = S_i R'_c R'_d = S_i R'_d R'_c, \tag{13.43}$$

where S_i (rather than S'_i) means the phase shifts ϕ_c and ϕ_d are zero. The unitary time evolution,

$$|\Omega_t\rangle = T^t|\Omega_0\rangle, \quad |\Omega_0\rangle = |0a,0\hat{c},0\hat{d}\rangle, \tag{13.44}$$

of an initial state $|\Omega_0\rangle$ in which the particle is at $[0a]$ and both detectors are in their ready states results in

$$
\begin{aligned}
|\Omega_4\rangle &= \alpha\,|4f,0\hat{c},0\hat{d}\rangle \\
&+ \tfrac{1}{2}\beta\big\{|4e,1\hat{c},0\hat{d}\rangle + |4f,1\hat{c},0\hat{d}\rangle - |4e,0\hat{c},1\hat{d}\rangle + |4f,0\hat{c},1\hat{d}\rangle\big\}
\end{aligned}
\tag{13.45}
$$

at $t = 4$; for any later time $|\Omega_t\rangle$ is given by the same expression with 4 replaced by t.

Consider a family of two-time histories with initial state $|\Omega_0\rangle$ at $t = 0$, and at $t = 4$ a decomposition of the identity in which each detector is in a pointer state (ready or triggered) and the particle emerges in either the e or the f channel. Consistency follows from the fact that there are only two times, and the probabilities can be computed using (13.45) as a pre-probability. There is a finite probability $|\alpha|^2$ that at $t = 4$ neither detector has detected the particle, and in this case it always emerges in the f channel. On the other hand, if the particle has been detected by the c detector, it will emerge with equal probability in either the e or the f channel, and the same is true if it has been detected by the d detector.

All of this agrees with Feynman's discussion of electrons passing through a double slit and illuminated by a weak light source. Emerging in the f channel rather than the e channel is what happens when no detectors are present inside the interferometer, and represents an interference effect. By contrast, detection of the particle in either arm washes out the interference effect, and the particle emerges with equal probability in either the e or the f channel. Note that the probability is zero that *both* detectors will detect the particle. This is what one would expect, since the particle cannot be both be *in* the c arm and *in* the d arm of the interferometer; quantum particles are never in two different places at the same time.

Additional complications arise when there is a weakly-coupled detector in only one arm, or when the numerical coefficients in (13.42) are different from those in (13.40). Sorting them out is best done using $T = S_i' R_c' R_d'$ or $T = S_i' R_c'$ in place of (13.43), and thinking about what happens when the phase shifts ϕ_c and ϕ_d are allowed to vary. Exploring this is left to the reader.

When weakly coupled detectors are present, what can we say about the particle *while it is inside the interferometer*? Again assume, for simplicity, that ϕ_c and ϕ_d are zero. There are many possible frameworks, and we shall only consider one example, a consistent family whose support consists of the five histories

$$
\begin{aligned}
&[\Omega_0]\odot[1c,0\hat{c},0\hat{d}]\odot[2c,0\hat{c},0\hat{d}]\odot[3c,1\hat{c},0\hat{d}]\odot[4e,1\hat{c},0\hat{d}],\\
&[\Omega_0]\odot[1c,0\hat{c},0\hat{d}]\odot[2c,0\hat{c},0\hat{d}]\odot[3c,1\hat{c},0\hat{d}]\odot[4f,1\hat{c},0\hat{d}],\\
&[\Omega_0]\odot[1d,0\hat{c},0\hat{d}]\odot[2d,0\hat{c},0\hat{d}]\odot[3d,0\hat{c},1\hat{d}]\odot[4e,0\hat{c},1\hat{d}],\\
&[\Omega_0]\odot[1d,0\hat{c},0\hat{d}]\odot[2d,0\hat{c},0\hat{d}]\odot[3d,0\hat{c},1\hat{d}]\odot[4f,0\hat{c},1\hat{d}],\\
&[\Omega_0]\odot[1\bar{a},0\hat{c},0\hat{d}]\odot[2\bar{a},0\hat{c},0\hat{d}]\odot[3\bar{a},0\hat{c},0\hat{d}]\odot[4f,0\hat{c},0\hat{d}].
\end{aligned}
\tag{13.46}
$$

(Consistency follows from the orthogonality of the final projectors, Sec. 11.3.) Using this family one can conclude that if at $t = 4$ the $\hat{c}$ detector has been triggered, the particle was earlier ($t = 1$, 2, or 3) in the c arm; if the $\hat{d}$ detector has been triggered, the particle was earlier in the d arm; and if neither detector has been triggered, the particle was earlier in a superposition state $|\bar{a}\rangle$. The corresponding statements for Feynman's double slit with a weak light source would be that if a photon scatters off an electron which has just passed through the slit system, then the electron previously passed through the slit indicated by the scattered photon, whereas if no photon scatters off the electron, it passed through the slit system in a coherent superposition.

While these results are not unreasonable, there is nonetheless something a bit odd going on. The projector $[1\bar{a}, 0\hat{c}, 0\hat{d}]$ at time $t = 1$ in the last history in (13.46) does not commute with the projectors at $t = 1$ in the other histories, even though the projectors for the histories themselves (on the history space $\breve{\mathcal{H}}$) do commute with each other, since their products are zero. This means that the Boolean algebra associated with (13.46) does not contain the projector $[1\bar{a}]_1$ for the particle to be in a coherent superposition state at the time $t = 1$, nor does it contain $[1c]_1$ or $[1d]_1$. Thus the events at $t = 1$, and also at $t = 2$ and $t = 3$, in these histories are *dependent* or *contextual* in the sense employed in Sec. 6.6 when discussing (6.55). Within the framework represented by (13.46), they only make sense when discussed together with certain later events; they depend on the later outcomes of the weak measurements in a sense which will be discussed in Ch. 14.