What Is Quantum Information?

Robert B. Griffiths
Carnegie-Mellon University
Pittsburgh, Pennsylvania

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- References (work by R. B. Griffiths)
  - "Nature and location of quantum information." Phys. Rev. A 66 (2002) 012311; quant-ph/0203058
  - "Channel kets, entangled states, and the location of quantum information," Phys. Rev. A 71 (2005) 042337; quant-ph/0409106
  - *Consistent Quantum Theory* (Cambridge 2002) http://quantum.phys.cmu.edu/

Introduction

- What is quantum information? Precede by:
  - What is information?
  - What is classical information theory?

- What is information? Example, newspaper
  - Symbolic representation of some situation
  - Symbols in newspaper *correlated* with situation
  - Information is *about* that situation

Classical Information Theory

- Shannon:
  - "Mathematical Theory of Communication" (1948)
  - One of major scientific developments of 20th century
  - Proposed *quantitative* measure of information
- Information entropy

$$H(X) = -\sum_i p_i \log p_i$$

  - Logarithmic measure of missing information
  - *Probabilistic* model: $\{p_i\}$ are probabilities
  - Applies to classical (macroscopic) signals
- Coding theorem: Bound on rate of transmission of information through noisy channel

Quantum Information Theory (QIT)

- Goal of QIT: “Quantize Shannon”
  - Extend Shannon’s ideas to domain where quantum effects are important
  - Find quantum counterpart of $H(X)$
- We live in a quantum world, so
  - QIT should be the *fundamental* info theory
  - Classical theory should emerge from QIT
  - Analogy: relativity theory $\rightarrow$ Newton for $v \ll c$

QIT: Current Status

- Enormous number of published papers
  - Does activity = understanding?
- Some topics of current interest:
  - Entanglement
  - Quantum channels
  - Error correction
  - Quantum computation
  - Decoherence
- Unifying principles have yet to emerge
  - At least, they are not yet widely recognized

QIT: Proposals

Published answers to question:
What is Quantum Information?

- Bennett and Shor (1998)
  - ◦ Qm $\leftrightarrow$ Cl info is like complex $\leftrightarrow$ real numbers
  - – Interesting analogy, but what are the details?
- Deutsch and Hayden (2000)
  - ◦ Technical idea using Heisenberg representation
  - – Causality, not information in Shannon sense
- Brukner and Zeilinger (2001)
  - ◦ Shannon ideas don't work in quantum domain

– But they do, if quantum probabilities correctly defined

- Caves, Fuchs, Schack (2002 and later)
  - ◦ Quantum wavefunction represents *our knowledge*
  - – Our knowledge of *what*?

QIT: Problem

- Quantizing Shannon faces a fundamental problem
  - Shannon theory based on *probabilities*
  - What *quantum* probabilities to put in formulas?
- Textbook (Copenhagen) quantum mechanics:
  - Probabilities $\leftrightarrow$ *measurement outcomes*
  - Measurement outcomes are macroscopic (classical)
  - Measurements *do not* reveal quantum properties
  - So provide no basis for *quantum* information
- Solution: Modernize the textbooks!
  - Measurements *do* reveal quantum properties
  - *Quantum* probabilities possible *if* one uses a *consistent* formulation of quantum theory!

Consistent Quantum Information

- Histories approach to quantum probabilities
  - ∘ Developed by Gell-Mann, Griffiths, Hartle, Omnès
  - ∘ Precise mathematical, logical rules
  - – Apply to quantum systems of any size
  - – Consistent results; always know what you're doing

- Naive quantum probability assignments
  - ∘ Result in paradoxes, mysteries, confusion

- Histories approach employs:
  - ∘ Many *frameworks* for quantum probabilities
  - ∘ Consistent probabilities in each framework
  - ∘ Cannot *combine* probabilities from *incompatible* frameworks

- Different incompatible frameworks ↔ different *types* / *kinds* / *species* of quantum information
  - ∘ Different species *cannot be combined*!

- Will use "multiple species" approach to discuss
  - ∘ "One bit" and ordinary (two bit) teleportation
  - ∘ Decoherence

Spin 1/2 Example

- Measure S_x using Stern-Gerlach
  - Result is $S_x = +1/2$ or $S_x = -1/2$ (units of $\hbar$)
  - Measurement *outcome* (pointer position) provides information about S_x *before* measurement took place.
  - Call this X *information* about the particle

- Measure S_z using Stern-Gerlach
  - Result is $S_z = +1/2$ or $S_z = -1/2$
  - Call this Z *information* about the particle

- X info and Z info are *incompatible*, different species, they cannot be combined.

- "$S_x = +1/2$ AND $S_z = +1/2$" is *meaningless*
  - Corresponds to nothing in Hilbert space
  - So quantum mechanics assigns it no meaning

- "$S_x = +1/2$ OR $S_z = +1/2$" is also meaningless

- "$S_x = +1/2$ AND $S_x = -1/2$" is meaningful, FALSE
  - Just one kind or species of information involved

- "$S_x = +1/2$ OR $S_x = -1/2$" meaningful and TRUE

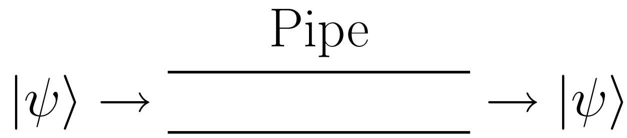
Classical Information

- Macroscopic object with angular momentum $\boldsymbol{L}$
  - One can measure L_x or L_z
  - Different *pieces* of information, *same species*
  - Can be combined in a meaningful way
  - "$L_x = 5$ Js AND $L_z = 7$ Js" makes sense

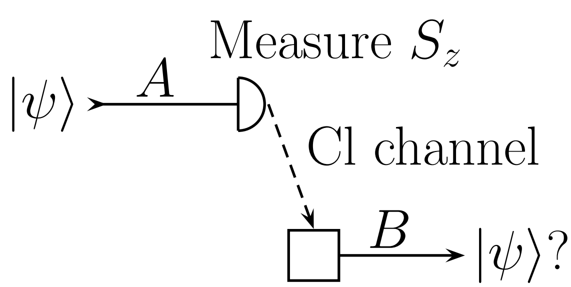
- We live in a quantum world!
  - Need *only one* quantum info species for macro world
  - By convention, this species is "classical information"
  - There is *no classical information* that is not some sort of *quantum information*.

Quantum Channel

- One-qubit (spin-half particle) quantum channel



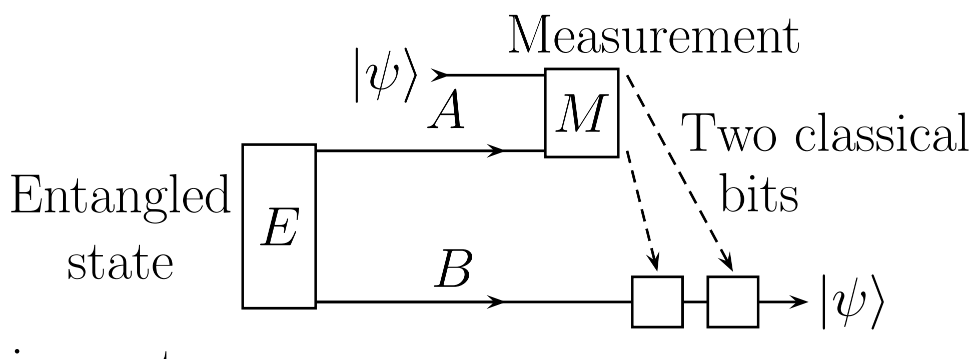
- Perfect channel: $|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$
  - Z info: $S_z = +\frac{1}{2}, -\frac{1}{2} \leftrightarrow |\psi_{\text{in}}\rangle = |0\rangle, |1\rangle$
  - X info: $S_x = +\frac{1}{2}, -\frac{1}{2} \leftrightarrow |\psi_{\text{in}}\rangle = |+\rangle, |-\rangle$
  - *Only one* species goes through at one time
  - *Any* species is correctly transmitted
- Long distance transmission problem



  - Can send Z information, but not X information
  - Alternative measurement: send X info, not Z
- Cannot make Qm channel using Cl channel
  - Cl channel transmits *only one* species of Qm info
  - Qm channel transmits *all* species

Teleportation

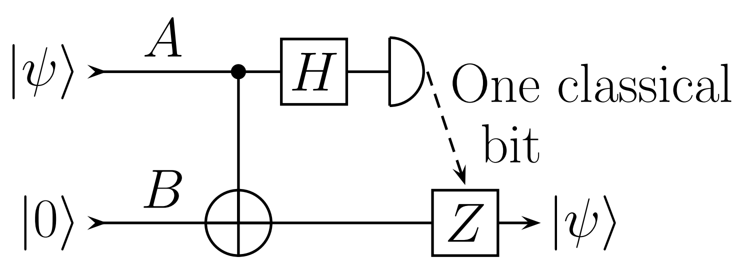
- Teleportation: Bennett et al. (1993)



- Requirements:
  - Shared entangled state already exists
  - Correlated measurement of two qubits
  - Two bits to send measurement outcome to B
  - Two unitary corrections by B

- Why *two* classical bits? Why not one? or three?

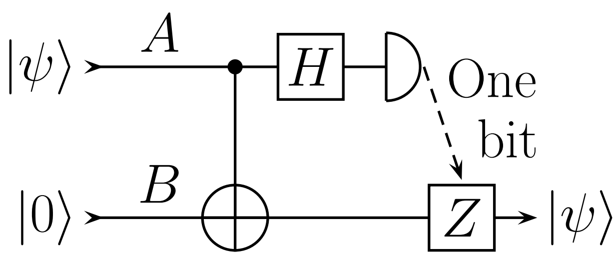
- One-bit Teleportation: Zhou et al. (2000)

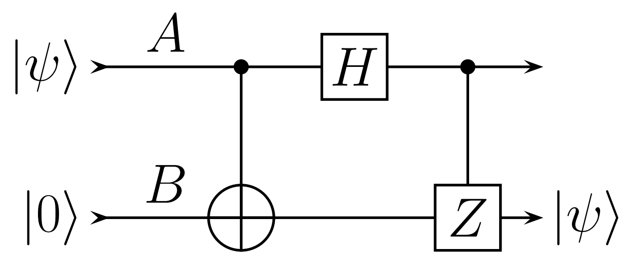


  - No entangled, state, only one classical bit, but
  - Requires nonlocal CNOT gate between A and B

One-Bit Teleportation: Z Information

- Circuit: Original and Quantized





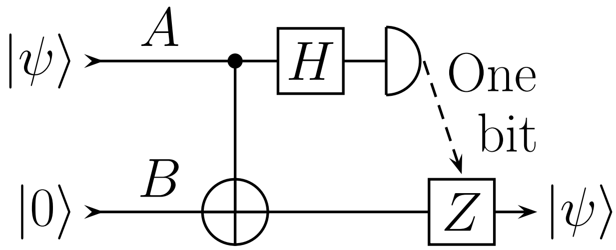
- Z information (S_z) about initial state of A qubit:
  - Z info = difference between initial $|0\rangle$ and $|1\rangle$
  - Is *copied* to B by the CNOT gate
  - Is unaffected by final Z gate:

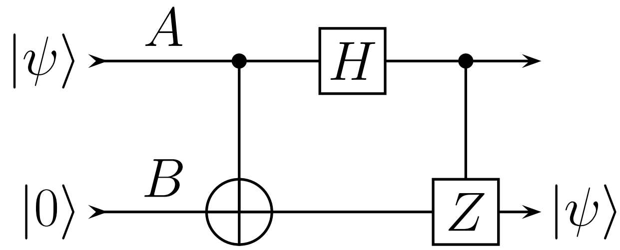
$$Z|0\rangle = |0\rangle, \quad Z|1\rangle = -|1\rangle$$

(phase -1 is unimportant for Z information)

- Conclusion: Z info arrives unaltered at output
  - Even if classical bit (or quantum counterpart) omitted

One-Bit Teleportation: X Information





- X information (S_x) about initial state of A qubit:
  - X info = difference between initial $|+\rangle$ and $|-\rangle$
- CNOT puts it in *correlation* between A and B qubits:

$$|+\rangle \rightarrow \tfrac{1}{2}\Big(|++\rangle + |--\rangle\Big), \quad S_{Ax} = S_{Bx}$$
$$|-\rangle \rightarrow \tfrac{1}{2}\Big(|+-\rangle + |-+\rangle\Big), \quad S_{Ax} = -S_{Bx}$$

  - X info *not* present in *individual* A, B qubits
- Hadamard H gate converts X to Z:
  - Case $|+\rangle \rightarrow |0\rangle$, no correction needed
  - Case $|-\rangle \rightarrow |1\rangle$, final Z gate changes $|+\rangle \leftrightarrow |-\rangle$
- Conclusion: X info arrives unaltered at output
  - Classical bit (or quantum counterpart) is essential
  - X info not in classical bit *by itself*

Information In Correlations

- Information in correlations is a classical, not a special quantum concept
- Illustration: C sends message by
  - Mailing colored slips of paper to A and B
  - Colors are red (R) or green (G)
  - Message 0: Same color (RR or GG) to A, B
  - Message 1: Different colors (RG or GR) to A, B
- Neither A nor B *individually* can read the message
  - Information of 0 vs. 1 is in *correlation* of colors

“Presence” Theorem

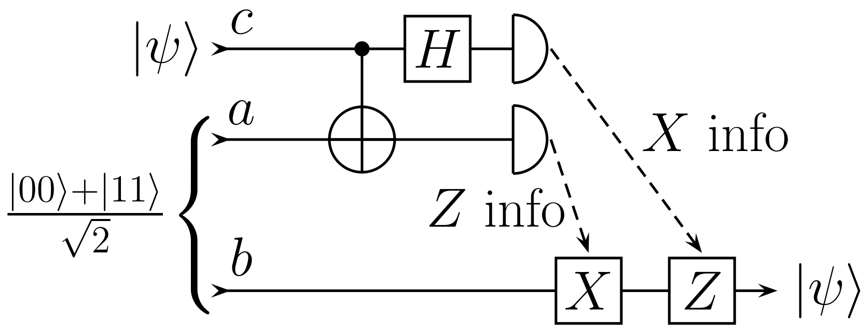
• One-bit teleportation works for Z info and X info
  ◦ What about Y info? Other species?
  ◦ We don't need to check them all because of the:

• Presence Theorem (qubits): If *any two incompatible* species of information are correctly transmitted from input to output, the same is true of *all* species.
  ◦ Theorem applies to noise-free transmission
  ◦ Refers to *two incompatible* species, so this is a *quantum information* theorem, no classical counterpart

• Generalization to d-dimensions: If *two* “sufficiently incompatible” species are correctly transmitted, *all* species are correctly transmitted (channel is perfect).
  ◦ Example: two orthonormal bases $\{|a_j\rangle\}$ and $\{|\bar{a}_k\rangle\}$, with $\langle a_j|\bar{a}_k\rangle \neq 0$ for all j, k, are sufficiently incompatible

Regular (2 Bit) Teleportation

- Circuit:



- One "classical" bit carries Z, the other X information
  - ◦ Use only "X" bit: will transmit X info
  - ◦ Use only "Z" bit: will transmit Z info
  - ◦ Quantization of circuit left as exercise

- Each species of information is in a *correlation* between the "classical" bit and the b qubit
  - ◦ Measuring classical bit tells one nothing
  - ◦ Measuring the b qubit tells one nothing

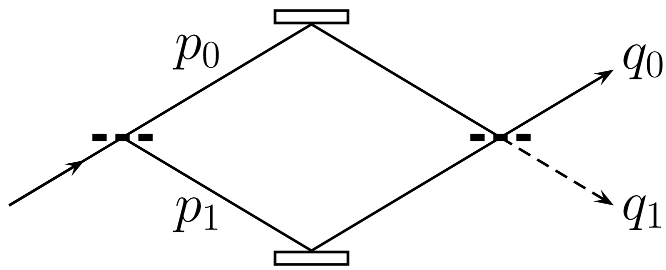
- Teleportation needs 2 classical bits because
  - ◦ There is *more than one* species of quantum info
  - ◦ If *two* species correctly transmitted, others follow
  - – So do not need three (or more) bits

- d-dimensional teleportation (qudit): same argument
  - ◦ Two incompatible species $\Rightarrow 2\log_2 d$ classical bits

Decoherence: Introduction

- Decoherence results when a quantum system interacts with its (quantum) environment
- Old perspective: Off-diagonal elements of density matrix go to zero
- New perspective (Zurek): *Information* flows from system to environment
- What can we learn using incompatible species of info?

Example: Interferometer



- No decoherence, particle initially in $|p+\rangle$

$$|p+\rangle := \big(|p_0\rangle + |p_1\rangle\big)/\sqrt{2} \rightarrow |q_0\rangle$$
$$|p-\rangle := \big(|p_0\rangle - |p_1\rangle\big)/\sqrt{2} \rightarrow |q_1\rangle$$

  ◦ Particle emerges in q_0, not q_1, because of *coherence*

- Decoherence:
  ◦ Which path, $|p_0\rangle$ vs $|p_1\rangle$, info $\rightarrow$ environment
  ◦ Particle emerges randomly in q_0 or q_1

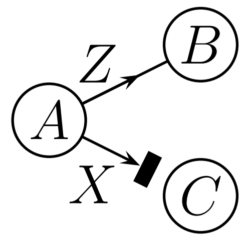
- Interpretation using different incompatible species:
  ◦ Z (which path) info: $|p_0\rangle$ vs $|p_1\rangle$
  ◦ X (which phase) info: $|p+\rangle$ vs $|p-\rangle$
  ◦ Decoherence means X (which phase) information has vanished when particle exits interferometer

- Z info in environment $\Rightarrow$ X info absent at output
  ◦ Consequence of Absence Theorem

"Absence" Theorem

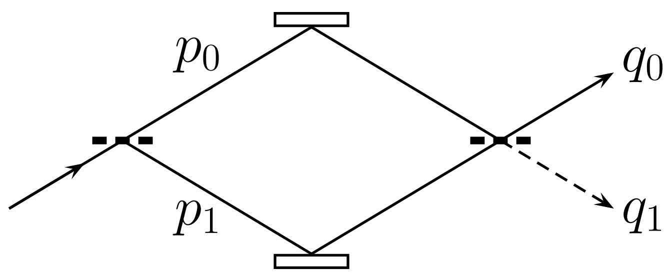
• Theorem. Three systems A, B, C. If Z info about A is *present* in B, then X info about A is *absent* from C.

◦ Two incompatible species; this is a *quantum* information theorem



◦ Present = perfectly present, Absent = perfectly absent

• Application to decoherence:



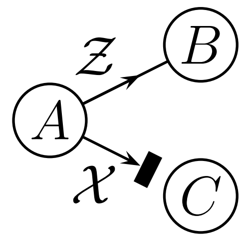
• If which path (Z) info about particle (A) entering interferometer is in the environment (B), coherent (X) info not present in particle (C) exiting interferometer, so there is *no interference*

◦ Particle at earlier (A) and later (C) times can be thought of as two systems when applying the theorem

General "Absence" Theorem

• Three systems: A, B, C. Dimension d of A arbitrary.
$\mathcal{Z} = \{|a_j\rangle\}$, $\mathcal{X} = \{|\bar{a}_k\rangle\}$ mutually unbiased bases of A.

• Theorem. If $\mathcal{Z}$ info about A is *present* in B, then $\mathcal{X}$ info about A is *absent* from C.



∘ Present = perfectly present, Absent = perfectly absent
∘ There may be better ways of wording the theorem

Decoherence: Conclusion

• If a particular species ("pointer basis") of information about the (earlier) state of a quantum system is available at some place in the environment, then other maximally-incompatible species of information about the same system will *not* be present at other places in the environment, or in the system itself.

◦ (Pace Zurek) It does *not* matter *how many* different places in the environment the information is located.

◦ It must be "clearly" present in (at least) one place.

◦ Generalization of Absence Theorem to *partial* presence or absence would be worthwhile.

Summary

• By distinguishing different incompatible species we can:
∘ Trace information flow in teleportation
– See why 2 classical bits are needed
– Or 1 classical bit for 1-bit teleportation
∘ Understand decoherence as a process in which spreading one species of information excludes others

• Open issues:
∘ Extend "Presence", "Absence" theorems to *partial* presence/absence
∘ Can information species be used to clarify asymptotic properties (channel capacities of various sorts)?