COURSE WEB PAGE: The following are identical
http://www.andrew.cmu.edu/course/33-445
http://quantum.phys.cmu.edu/quad/

READING: Sources
Townsend = A Modern Approach to Quantum Mechanics, 2d ed
CQT = Griffiths, Consistent Quantum Theory. Individual chapters are available at:
http://quantum.phys.cmu.edu/CQT/
HSQM = “Hilbert Space Quantum Mechanics” on course web page
PROBS = “Probabilities” on course web page

READING: Topics
Spin half: Townsend, Chs. 1 and 2; HSQM.
Hilbert space: Townsend, Chs. 1 and 2; HSQM. CQT Ch. 3 summarizes the linear algebra in Dirac notation needed for quantum mechanics
Probabilities: CQT Ch. 5; PROBS. For classical probability theory see any introductory textbook for probability and statistics. You may find something useful at Wikipedia, though the quantum probabilities article is not very helpful.

EXERCISES:
1. Consider the following collection of kets for a spin-half particle
   (i): \( |\psi_1\rangle = 2|z^+\rangle \), (ii): \( |\psi_2\rangle = |z^+\rangle - (i/2)|z^-\rangle \), (iii): \( |\psi_3\rangle = 2i|z^+\rangle + |z^-\rangle \), (iv): \( |\psi_4\rangle = (1/\sqrt{3})|z^+\rangle + (\sqrt{2/3})|z^-\rangle \).
   a) Write down the corresponding bra vectors as linear combinations of \( \langle z^+ | \) and \( \langle z^- | \).
   b) Which kets are normalized? For those which are not, find a normalized version \( |\bar{\psi}_j\rangle \).
   c) Explain why your answers to (b) are not unique.
   d) Evaluate the following inner products:
      \( \langle \psi_1 | \psi_4 \rangle \), \( \langle \psi_2 | \psi_3 \rangle \), \( \langle \psi_3 | \psi_2 \rangle \), \( \langle \psi_3 | \psi_4 \rangle \)
   e) Consider all six pairs \{\( |\psi_j\rangle, |\psi_k\rangle \), \( j < k \), and state whether they represent the same physical property, two incompatible properties, or two mutually-exclusive properties.
   f) For each \( |\psi_j\rangle \) find a normalized ket which is orthogonal to it.
2. Consider for a Hilbert space of dimension \( d = 3 \) the kets
   \[ |v\rangle = \begin{pmatrix} 1 \\ 2 \\ -i \end{pmatrix}, \quad |w\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \]
   where \( a, b, \) and \( c \) are complex numbers.
   a) Write down a condition on \( a, b, \) and \( c \) which will ensure that \( |w\rangle \) and \( |v\rangle \) are orthogonal, \( \langle w | v \rangle = 0 \).
   b) Find two linearly independent kets \( |w_1\rangle \) and \( |w_2\rangle \) such that \( \langle w_1 | v \rangle = \langle w_2 | v \rangle = 0 \).
   c) [This part will not be graded, but you are strongly urged to try and find an answer.] Explain why you cannot find three linearly independent kets \( |w_1\rangle, |w_2\rangle, \) and \( |w_3\rangle \) which are all orthogonal to \( |v\rangle \).
3. The T-die (plural T-dice) is a solid tetrahedron with \( s = 1, 2, 3, \) and 4 spots on its four faces. When it is rolled it is the face next to the table, thus invisible from above, that indicates the value of \( s \). E.g., “It came down 3” in the case where the visible faces show 1, 2, and 4 dots.

a) Alice and Bob play a game in which Bob pays Alice $2 if \( s = 2 \) or 4. If \( s = 1 \) Alice pays Bob $1, and if \( s = 3 \) Alice pays Bob $3. Construct a random variable \( W(s) \) which is the amount Alice wins (\( W > 0 \)) or loses \( W < 0 \) following a roll of the T-die. Assuming equal probabilities for all four faces, find \( \langle W \rangle \) and \( \Delta W \), the average and the standard deviation.

b) However, this T-die is unsymmetrical and some experimentation shows that in fact the probabilities for \( s \) are given by \( p_s = 0.27 \) for \( s = 1, 2, \) and 3. What is \( p_4 \)? Again compute \( \langle W \rangle \) and \( \Delta W \).

c) The game given the probabilities in (b) is no longer fair. Indicate how it can be made fair by changing just one of three amounts ($1, $2, or $3) specified in (a). Make one of these changes, write down the new \( W \) function, call it \( \bar{W} \). Check that \( \langle \bar{W} \rangle = 0 \), and compute \( \Delta \bar{W} \).

d) After class discussion.

Thomas: “Wouldn’t it be simpler to use a sample space in which there are just three possibilities, not four? These would be: \( t = 1 \) the same as \( s = 1 \), \( t = 3 \) the same as \( s = 3 \), but \( t = 2 \) would stand for both \( s = 2 \) and \( s = 4 \).”

Ursula: “If that’s possible I propose going further: a sample space with two elements: \( u = 1 \) would stand for both \( s = 1 \) and \( s = 3 \), and \( u = 2 \) for both \( s = 2 \) and \( s = 4 \).”

Vincent: “Why not allow some overlap? If \( s = 1 \) or \( s = 2 \) or \( s = 3 \), I set \( v = 1 \); if \( s = 2 \) or \( s = 4 \), I set \( v = 2 \). Isn’t my sample space as good as any of your proposals?”

William: “What I prefer is a two element sample space, where \( w = 1 \) is stands for \( s = 1 \), and \( w = 2 \) for \( s = 2 \) and \( s = 4 \).”

It is now your turn to join the discussion. What do you think of these proposals? Are any of them satisfactory? Why or why not?

c) [This part will not be graded, but you are urged to think about it.] Before the game is played, Alice and Bob agree that it will stop as soon as the total amount that one or the other has won (or lost) exceeds $20. Make a rough estimate of a typical number of rolls of the T-die before this agreement will bring the game to an end. [Hint. The math books tell us that if there are \( N \) independent trials the variance of the sum of the results increases proportional to \( N \).]

4. Problem 1.3 parts (b) and (c) together with Problem 1.4 in Townsend. Check that \( \Delta S_z \) and \( \Delta S_x \) are zero and achieve their maximum values at the values of \( \theta \) and \( \phi \) you would expect.