Instantaneous Nonlocal Measurements

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References

- Entanglement consumption of instantaneous nonlocal quantum measurements.
  arXiv:1004.0865v1

- Instantaneous measurement of non-local variables.

- Measurements of semi-local and non-maximally entangled states.
Structure of the talk

- Definition of problem.
- Motivations.
- Simple schemes on two qubits.
- The Vaidman scheme. (using teleportation)
- Finite entanglement consumption scheme. (better use of teleportation)
Definition of instantaneous nonlocal measurements

- Bipartite nonlocal quantum measurement: a measurement of a Hermitian observable on a bipartite system. The final state might be disturbed (destroyed or altered), i.e. might not be one of the eigenstates.
- Special case: state verification (verifies if the state was $|\psi\rangle$). Probability of “yes” is given by $|\langle\psi|\eta\rangle|^2$, where $|\eta\rangle$ is an arbitrary initial state. Corresponds to a coarse-grained measurement. The final state could be disturbed.
- “Instantaneous” means no classical communication is allowed except at classical postprocessing. Both parties perform local operations using shared entanglement, and then the local measurement outcomes are sent to a third party $C$ to recover the result of the nonlocal measurement.
**Motivations**

- Understanding quantum mechanics: nonlocal variables as observables.
- Understanding the relationship between classical and quantum nonlocal correlations.
- Time as a resource. Applications in “fast” protocols for nonlocal unitaries.
The nonlocal unitary related to a nonlocal measurement

- For any nonlocal observable $O$, there is a (nonlocal) unitary $U$ that maps the eigenstates of the observable $O$ to a product basis.
- The $U$ corresponding to a Bell measurement is the CNOT gate (up to local unitaries).
- A nonlocal measurement protocol in general corresponds to a quantum channel on $AB$ followed by local measurements on $A,B$, so it could use less entanglement than the entanglement cost of $U$. 
**Instantaneous nonlocal Bell measurement**

*Figure:* Instantaneous nonlocal Bell measurement protocol that uses 1 ebit. Result is in the correlations between the local measurement outcomes.
Instantaneous measurement of any two-qubit observable

- The case of controlled-Hadamard gates. Needs 1 ebit.
- The case of general controlled gates.
- More general cases.
The Vaidman scheme

- Works for any observable. Based on the idea of sending the system back and forth using partial teleportation (without sending local measurement outcomes).
- Step 1: Bob teleports his system to Alice. She applies $U$ to the combined system that transforms the eigenstates of the observable to the product $Z$ basis.
- Step 2: Alice then teleports the combined system back to Bob. If Bob’s previous measurement outcome $n = 1$, he measures in the standard basis and terminates. Otherwise, he teleports the combined system to Alice using a copy of the entangled ancilla dependent on the value of $n$.
- Step 3: Alice then does a different unitary on each copy that corrects $U$ using the information about $n$ and her previous measurement result $k$. Go to step 2. The process terminates when Bob’s previous measurement outcome was 1.
The Vaidman scheme: simplest case

Figure: The case $n = 1$ in Vaidman’s protocol (on two qubits). $T$ denotes the gates in teleportation.
The Vaidman scheme: more complex cases

Figure: The cases $n \in \{2, 3, 4\}$ in Vaidman’s protocol (on two qubits). Bob only teleports using the entangled ancilla labeled by $j = n + 1$, but Alice performs the operation in the red box for each $j \in \{3, 4, 5\}$. $U_j\vec{\sigma}_k U\sigma_n = U.$
The Vaidman scheme: discussions

- Bob terminates when he gets measurement result 1 (no correction needed for teleportation). Alice has no termination condition. Thus the worst-case entanglement cost is unbounded.
- If we allow Alice to terminate after $M$ rounds, and denote the corresponding probability of failure by $q$, then the entanglement cost is exponential in $M$, but is a polynomial function of $1/q$. For two-qubit measurements, the entanglement cost $\sim 15^M \sim 10^{-6} \cdot (1/q)^{42}$, e.g. $\sim 10^{16}$ ebits for $q = 0.3$, and $\sim 10^{36}$ ebits for $q = 0.1$.
- The case that each party has $d$ qubits is similar. The protocol can be generalized to any number of parties.
Finite consumption scheme

- **Main idea:** for the unitary $U$ that maps the eigenstates of the observable to the product $Z$ basis, decompose it into one or two-qubit Pauli rotation gates.
- **Pauli rotation:**
  \[ R_j(\theta) = \exp(-i\theta\sigma_j/2) = \cos(\theta/2)I - i \sin(\theta/2)\sigma_j, \]
  where $\sigma_j$ is a tensor product of one-qubit Pauli operators.
- **Basic component of the protocol:** Pauli rotation chain that implements $R_j(\theta)$. The chain consists of a sequence of teleportation channels in which the entire system is teleported back and forth.
Finite consumption scheme – two-qubit gate example 1

Figure: Instantaneous measurement with $U = R(\pi/2) = \exp(-i\frac{\pi}{4}\sigma_x \otimes \sigma_x)$.  

\[ U = R(\pi/2) = \exp(-i\frac{\pi}{4}\sigma_x \otimes \sigma_x). \]
**Finite consumption scheme – two-qubit gate example 2**

*Figure:* Instantaneous measurement with

\[ U = R(\theta) = \exp(-i \frac{\theta}{2} \sigma_x \otimes \sigma_x) \].

The local unitary corrections (apart from the Pauli operators) are \( R(\theta), R(2\theta), R(4\theta), \ldots \).
Finite consumption scheme – more general gates

- Concatenation of rotation chains: the entanglement costs for sequential unitaries do not simply add up, because of the requirement of no-communication.
- Average entanglement consumption for one Pauli rotation chain on 1 qubit: 5 ebits. For 2-4 chains concatenated: 20, 59, 156 ebits respectively. Entanglement cost grows exponentially with the number of chains.
State verification on two qubits

- The protocol in fact performs an orthogonal nonlocal measurement of an equally entangled basis (can be mapped to the quadratic Gauss sum type).
- For general states on two qubits, needs 6 ebits on average. But needs less entanglement for states with binary angles.
The average entanglement cost for the most general two-qubit measurement is 787 ebits.

Entanglement cost for multi-qubit measurements: \( U \) is decomposed into an exponential number of Pauli rotation gates (correspond to rotation chains in the protocol), thus the average entanglement cost is doubly-exponential in the number of qubits.
Relationship to nonlocal unitary protocols

- By teleporting at the end, the finite consumption scheme of nonlocal measurement can be turned into a fast protocol for doing nonlocal unitaries.
- Comparison of entanglement cost of the two approaches for some special unitaries: average and worst cases.
Summary

- Introduced the instantaneous nonlocal measurements.
- Discussed protocols for doing them: The Groisman and Reznik scheme, the Vaidman scheme and the finite consumption scheme.
- Discussed possible relationship to the nonlocal unitaries protocols.