Quantum Information Seminar/Workshop

Summary for 29 June 2006: Robert Griffiths and Hsun-Hsien Chang

(Version of 7/13/06)

Abstract

We demonstrate that any one-qubit gate can be simulated in the cluster-state model of computation. We also show that the controlled-phase gate can be efficiently simulated, even if the gate acts on non-neighboring qubits. Since these two sets of gates are universal for quantum computation, and gate simulations can be concatenated (Sec. II.D. of [2]), the cluster-state model is capable of efficiently simulating any quantum circuit.

1 One-qubit gates

Any one-qubit unitary U can be written in the following form:

$$U = Z_\gamma X_\beta Z_\alpha, \tag{1}$$

where

$$\begin{aligned} Z_\alpha &= \exp(-i\alpha Z/2), \\ X_\beta &= \exp(-i\beta X/2). \end{aligned} \tag{2}$$

We first show that one-qubit gates of the form $X_\beta Z_\alpha$ can be implemented (up to some Pauli factors) with the following circuit:

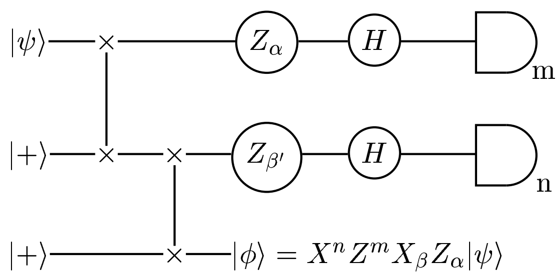

Figure 1: Simulating one-qubit gates of the form $X^n Z^m X_\beta Z_\alpha$.

Note that in the first row of the circuit in Fig. 1, the sequence of gates Z_α and H, and measurement in the Z basis is actually implemented by a measurement in the eigenbasis of X_α. Similarly, for the second qubit, a measurement in the eigenbasis of $X_{\beta'}$ is carried out. $\beta' = (-1)^m\beta$ is adjusted according to the result of the measurment on the first qubit.

In order to prove that the circuit above does yield the desired state $|\phi\rangle$, we consider its atemporal diagram, as shown in Fig. 2.

We need to apply the result in Sec. 3 of the notes of the last seminar. Considering the top half of the diagram, a state $X^m H Z_\alpha|\psi\rangle$ is transmitted from above to the middle Δ object. Using the same result again, we get that the final output of the third qubit is $X^n H Z_{\beta'}(X^m H Z_\alpha)|\psi\rangle$, which is equal to $X^n Z^m X_\beta Z_\alpha|\psi\rangle$.

The more general one-qubit unitary in (1) can be implemented in similar fashion. However, we can make the computation simpler by using (almost only) gates of the form $X_\beta Z_\alpha$ and controlled-phase gates. If, for example, two one-qubit gates of the form (1) precede and follow a controlled-phase gate, the last Z gate preceding the controlled-phase can be moved through the controlled-phase and combined with the first Z gate that follows, reducing the first of the gates to the $X_\beta Z_\alpha$ form.

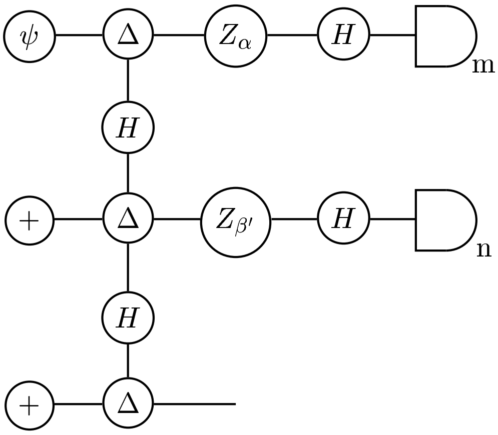

Figure 2: The atemporal form of the circuit in Fig. 1

In the discussions above, we have started with an arbitrary one-qubit state $|\psi\rangle$. But we can get $|\psi\rangle$ by applying a one-qubit unitary (with the method above) on the $|+\rangle$ state. That is why only the $|+\rangle$ state is needed in the initialization step of cluster-state computation.

2 Controlled-phase (CP) gates

To do universal quantum computation, we need to simulate an entangling gate on cluster states. The following figure shows a method to implement a controlled-phase gate on two qubits which are separated by two other qubits in a cluster state.

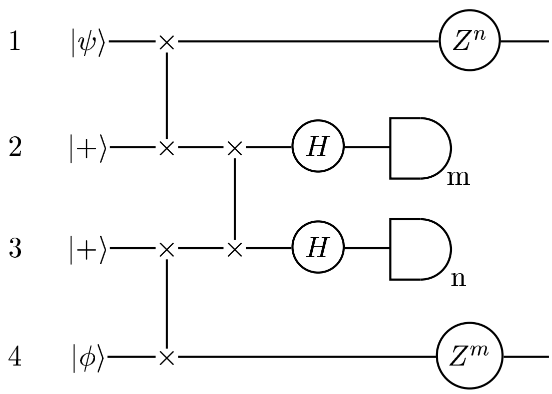

Figure 3: Simulating the controlled-phase gate (on qubits 1 and 4).

With the help of the diagram equalities introduced in the last seminar, it is not difficult to see that the circuit in Fig. 3 does simulate the controlled-phase gate.

Another way to realize controlled-phase gates is simply using the **CP** gates that were carried out in the preparation of the cluster state. This means to execute the **CP** gate before some other gates and measurements, which act on different qubits from those the **CP** gate acts on. Such exchange of time order makes no difference in the result of the computation.

The problem left is to carry out **CP** gates between qubits that are far away from each other. We can do this by transporting the two qubits to neighboring positions by a series of **SWAP** gates. The **SWAP** gate can be implemented with three **CNOT** gates, which are equivalent to **CP** gates under local unitaries. There may be other ways to do this. Raussendorf *et al* have designed a scheme to realize **CNOT** gates between non-neighboring qubits (Sec. IV.C. of [2]).

References

[1] R. Raussendorf and H.J. Briegel. A one-way quantum computer. *Phys. Rev. Lett.*, 86, 5188-5191(2001).

[2] R. Raussendorf, D.E. Browne, H.J. Briegel. Measurement-based quantum computation with cluster states. *Phys. Rev. A*, 68, 022312(2003). arXiv:quant-ph/0301052.

[3] M. A. Nielsen. Journal club notes on the cluster state model of quantum computation. http://www.qinfo.org/qc-by-measurement/cluster-state.pdf.

[4] M. A. Nielsen. Cluster-state quantum computation. arXiv:quant-ph/0504097.

[5] M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.J. Briegel. Entanglement in Graph States and its Applications. arXiv:quant-ph/0602096.

[6] D. Schlingemann, R.F. Werner. *Phys. Rev. A*, 65, 012308(2002). arXiv:quant-ph/0012111.

[7] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

[8] Robert B. Griffiths, Shengjun Wu, Li Yu, Scott M. Cohen. Atemporal diagrams for quantum circuits. *Phys. Rev. A*, 73, 052309(2006). arXiv:quant-ph/0507215.