Quantum Information Seminar/Workshop

Summary for 29 June 2006: Robert Griffiths and Hsun-Hsien Chang
(Version of 7/13/06)

Abstract

We demonstrate that any one-qubit gate can be simulated in the cluster-state model of computation. We also show that the controlled-phase gate can be efficiently simulated, even if the gate acts on non-neighboring qubits. Since these two sets of gates are universal for quantum computation, and gate simulations can be concatenated (Sec. II.D. of [2]), the cluster-state model is capable of efficiently simulating any quantum circuit.

1 One-qubit gates

Any one-qubit unitary $U$ can be written in the following form:

$$U = Z_\gamma X_\beta Z_\alpha,$$

where

$$Z_\alpha = \exp(-i\alpha Z/2),$$

$$X_\beta = \exp(-i\beta X/2).$$

We first show that one-qubit gates of the form $X_\beta Z_\alpha$ can be implemented (up to some Pauli factors) with the following circuit:

![Figure 1: Simulating one-qubit gates of the form $X^n Z^m X_\beta Z_\alpha$.](image)

Note that in the first row of the circuit in Fig. 1, the sequence of gates $Z_\alpha$ and $H$, and measurement in the $Z$ basis is actually implemented by a measurement in the eigenbasis of $X_\alpha$. Similarly, for the second qubit, a measurement in the eigenbasis of $X_\beta$ is carried out. $\beta' = (-1)^m \beta$ is adjusted according to the result of the measurement on the first qubit.

In order to prove that the circuit above does yield the desired state $|\psi\rangle$, we consider its atemporal diagram, as shown in Fig. 2.

We need to apply the result in Sec. 3 of the notes of the last seminar. Considering the top half of the diagram, a state $X^n H Z_\alpha \psi$ is transmitted from above to the middle $\Delta$ object. Using the same result again, we get that the final output of the third qubit is $X^n H Z_\beta' (X^n H Z_\alpha) |\psi\rangle$, which is equal to $X^n Z^m X_\beta Z_\alpha |\psi\rangle$.

The more general one-qubit unitary in (1) can be implemented in similar fashion. However, we can make the computation simpler by using (almost only) gates of the form $X_\beta Z_\alpha$ and controlled-phase gates. If, for example, two one-qubit gates of the form (1) precede and follow a controlled-phase gate, the last $Z$ gate preceding the controlled-phase can be moved through the controlled-phase and combined with the first $Z$ gate that follows, reducing the first of the gates to the $X_\beta Z_\alpha$ form.
In the discussions above, we have started with an arbitrary one-qubit state $|\psi\rangle$. But we can get $|\psi\rangle$ by applying a one-qubit unitary (with the method above) on the $|+\rangle$ state. That is why only the $|+\rangle$ state is needed in the initialization step of cluster-state computation.

2 Controlled-phase (CP) gates

To do universal quantum computation, we need to simulate an entangling gate on cluster states. The following figure shows a method to implement a controlled-phase gate on two qubits which are separated by two other qubits in a cluster state.

With the help of the diagram equalities introduced in the last seminar, it is not difficult to see that the circuit in Fig. 3 does simulate the controlled-phase gate.

Another way to realize controlled-phase gates is simply using the CP gates that were carried out in the preparation of the cluster state. This means to execute the CP gate before some other gates and measurements, which act on different qubits from those the CP gate acts on. Such exchange of time order makes no difference in the result of the computation.

The problem left is to carry out CP gates between qubits that are far away from each other. We can do this by transporting the two qubits to neighboring positions by a series of SWAP gates. The SWAP gate can be implemented with three CNOT gates, which are equivalent to CP gates.
under local unitaries. There may be other ways to do this. Raussendorf et al have designed a scheme to realize CNOT gates between non-neighboring qubits (Sec. IV.C of [2]).

References


