Quantum Information Seminar/Workshop

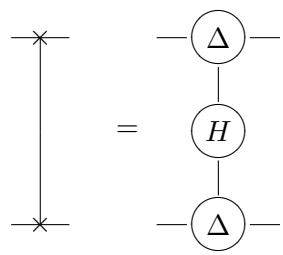
Summary for 22 June 2006: Robert Griffiths and Hsun-Hsien Chang
(Version of 7/7/06)

Abstract

In the last seminar, we introduced the concept of graph states, and presented an atemporal diagram for the two-qubit controlled-phase gate. This time we show that atemporal diagrams[6] can help us simplify some quantum circuits involving graph states. As an application, we analyze some basic operations for cluster-state quantum computation with atemporal diagrams.

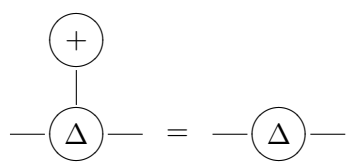
1 Some circuit equalities

First, we recall the following circuit equality for the controlled-phase gate (see Fig. 2 of the notes of the last seminar)



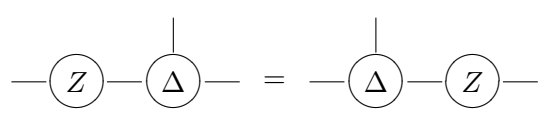
(1)

When the + state is linked to a Δ object, the + state and the line connecting to it can be eliminated:



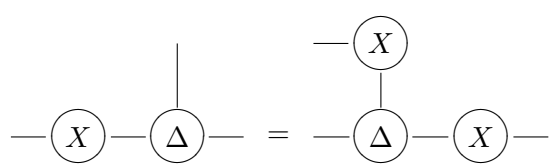
(2)

The Z gate can be moved freely through the Δ object and placed on one of the other legs:



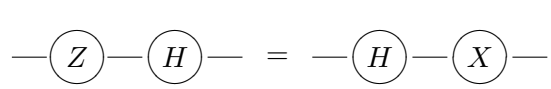
(3)

To remove an X gate adjacent to the Δ object, an X gate should be placed on *each* of the other legs of the Δ object:



(4)

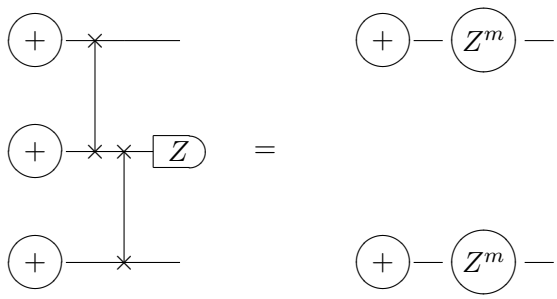
The Z gate becomes an X gate if it is moved through a H gate (and the X gate becomes a Z gate if it is moved through a H gate):



(5)

2 Erasing one node in a cluster state

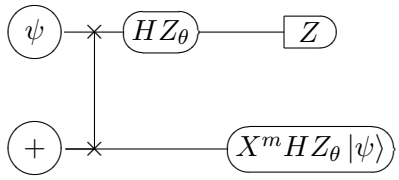
Now we show that a qubit can be removed from a cluster state. This helps a lot in the design of quantum computation schemes using cluster states. In the following diagram, the middle qubit (and the edges connecting it to the top and bottom qubits) could be removed. The method is to measure the qubit in the Z basis. If the result indicates that it was in the state $|0\rangle$ ($m = 0$ in the diagram), do nothing; otherwise ($m = 1$), apply a Z gate to each of its neighbors. This can be checked using the circuit equalities in Sec. 1.



(6)

3 Simulating one-qubit gate with measurement

One-qubit gates of the form HZ_θ can be implemented by doing measurement in some suitable basis, and then some adjustment depending on the measurement outcome. The following diagram is the atemporal form of Eq. (12) in [2].



(7)

References

[1] R. Raussendorf and H. J. Briegel. A one-way quantum computer. *Phys. Rev. Lett.*, 86, 5188-5191(2001).

[2] M. A. Nielsen. Cluster-state quantum computation. arXiv:quant-ph/0504097.

[3] M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.-J. Briegel. Entanglement in Graph States and its Applications. arXiv:quant-ph/0602096.

[4] D. Schlingemann, R.F. Werner. *Phys. Rev. A*, 65, 012308(2002). arXiv:quant-ph/0012111.

[5] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

[6] Robert B. Griffiths, Shengjun Wu, Li Yu, Scott M. Cohen. Atemporal diagrams for quantum circuits. *Phys. Rev. A*, 73, 052309(2006). arXiv:quant-ph/0507215.