Quantum Information Seminar/Workshop

Summary for 15 June 2006: Robert Griffiths and Hsun-Hsien Chang
(Version of 6/21/06)

Cluster-state quantum computation [1, 2] is a model of quantum computation in which a sequence of single-qubit measurements are applied to a fixed quantum state known as a *cluster state*, which is a *graph state* [3] on a lattice structure. The graph state is also useful in constructing quantum error-correction codes [4].

A graph state is produced by starting with a qubit for each vertex of the graph in the $|+\rangle$ (or, possibly, $|-\rangle$) state, and then applying a sequence of controlled-phase gates for those pairs of qubits which are neighbors in the sense that their vertices are connected by an (undirected) edge in the graph. The preparation of a two-qubit graph state is shown in Fig. 1.

Note that the order of applying the controlled-phase (CP) gates does not matter. These gates commute with each other. This can be seen by writing them in terms of Pauli gates, where Z_j is σ_z applied to qubit j, e.g.,

$$CP_{12} = (I + Z_1 + Z_2 - Z_1 Z_2)/2.$$

Of course, the Z_j all commute with each other. Another way to look at the controlled-phase gate is to use an atemporal diagram, Fig. 2.

Graph states have only real coefficients when expanded in the computational basis, so can be thought of as elements in a real Hilbert space; however, the same cannot be said for the generalization in which each node is not a qubit, but a qudit (d-level system) with $d \geq 3$.

Classical linear codes can be expressed in graphical form, by thinking of the coding matrix G and the syndrome matrix H, see Sec. 10.4 of [5], as adjacency matrices of certain graphs. (The adjacency matrix contains a 1 at the location jk if there is an edge between vertex j and vertex k, and a 0 at other positions.) Similar graphs can be used for quantum codes, e.g., the five-qubit code (p. 469 of [5])—this topic to be continued in later seminars.

References

[1] R. Raussendorf and H. J. Briegel. A one-way quantum computer. *Phys. Rev. Lett.*, 86, 5188-5191(2001).

[2] M. A. Nielsen. Cluster-state quantum computation. arXiv:quant-ph/0504097.

[3] M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.-J. Briegel. Entanglement in Graph States and its Applications. arXiv:quant-ph/0602096.

[4] D. Schlingemann, R.F. Werner. *Phys. Rev. A*, 65, 012308(2002). arXiv:quant-ph/0012111.

[5] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

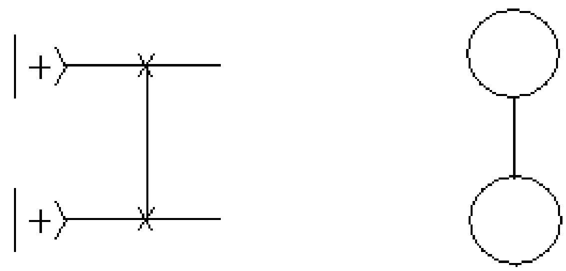

Figure 1: The preparation circuit and the resulting two-qubit graph state. The qubits are both initially in the state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. A controlled-phase gate is performed. The resulting state on these two qubits is $(|00\rangle + |01\rangle + |10\rangle - |11\rangle)/2 = (|0\rangle \otimes |+\rangle + |1\rangle \otimes |-\rangle)/\sqrt{2}$, thus it is maximally entangled.

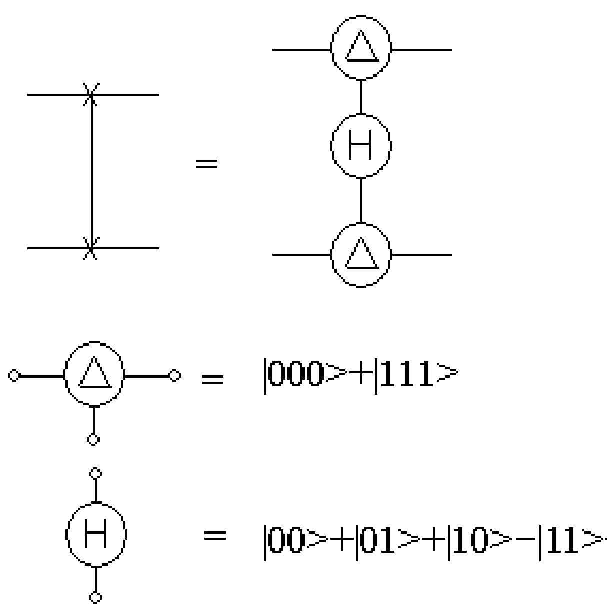

Figure 2: Another representation of the controlled-phase gate