Toy Model of Time-Delay Interference

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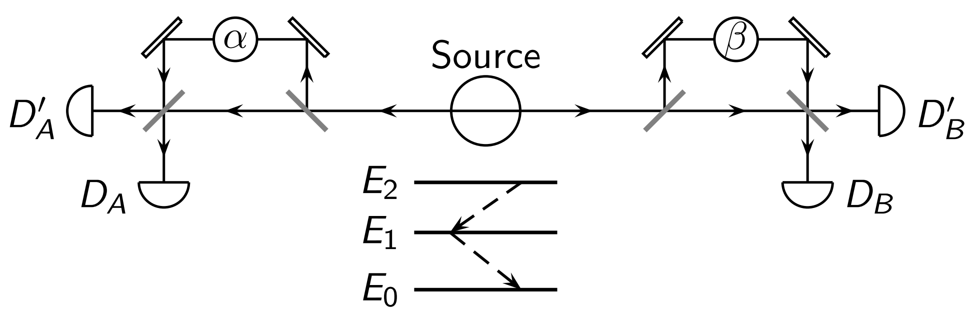
- References:

  ○ J. D. Franson, "Bell inequality for position and time", Phys. Rev. Lett. 62 (1989) 2205-2208.

  ○ R. B. Griffiths, *Consistent Quantum Theory* (Cambridge 2002); http://quantum.phys.cmu.edu/CQT/

Franson Interferometer I



- Photons emitted with energies $E_2 - E_1$ and $E_1 - E_0$

  ○ One goes to the left and one to the right

  ○ E_2: long lifetime; E_1: short lifetime

- Unbalanced Mach-Zehnder interferometers: Short path & long path

  ○ Phase shifs α and β in the longer arms

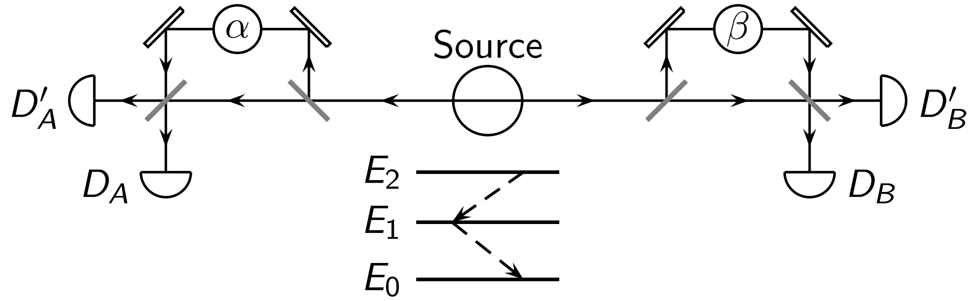
- Look for coincidences in detection:

D_A and D_B simultaneous, or D'_A and D_B simultaneous, etc.

  ○ Note that D_A and D'_A will *not* be simultaenous; likewise D_B and D'_B

- Original Franson paper: source = three-level atoms. More

Franson Interferometer II



• Photon source in original Franson paper: three-level atoms. More recent: down conversion using pulsed pump laser.

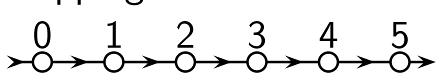
• Original paper (1989) had title:
"Bell inequality for position and time"

◦ Idea: this could test the Bell inequality (1964)

◦ Test had previously been carried out by Aspect (1981)

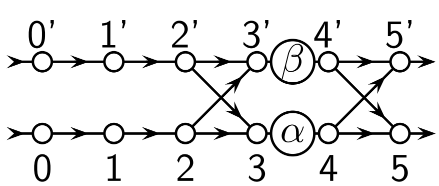
Toy Model Mach-Zehnder Interferometer I

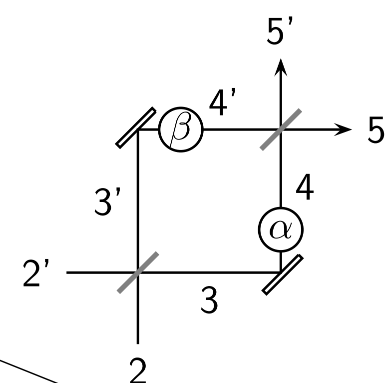
Hopping model



$S|m\rangle = |m+1\rangle$

$|0\rangle \to |1\rangle \to |2\rangle \to |3\rangle \to \cdots$





$S|m\rangle = |m+1\rangle \quad S|m'\rangle = |(m+1)'\rangle$

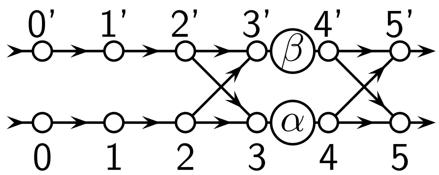
$\sqrt{2}S|2\rangle = |3\rangle + |3'\rangle \quad \sqrt{2}S|2'\rangle = |3\rangle - |3'\rangle$

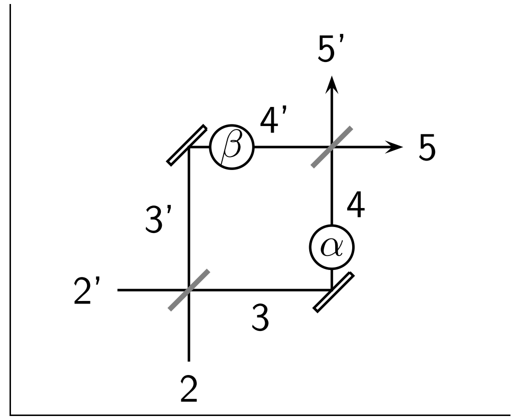
$S|3\rangle = \exp(i\alpha)|4\rangle \quad S|3'\rangle = \exp(i\beta)|4'\rangle$

$\sqrt{2}S|4\rangle = |5\rangle + |5'\rangle \quad \sqrt{2}S|4'\rangle = |5\rangle - |5'\rangle$

$$|2\rangle \to \frac{|3\rangle + |3'\rangle}{\sqrt{2}} \to \frac{e^{i\alpha}|4\rangle + e^{i\beta}|4'\rangle}{\sqrt{2}} \to \frac{(e^{i\alpha} + e^{i\beta})|5\rangle + (e^{i\alpha} - e^{i\beta})|5'\rangle}{2}$$

Toy Mach-Zehnder Interferometer II





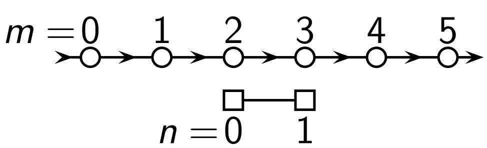
$$|2\rangle \to \frac{|3\rangle + |3'\rangle}{\sqrt{2}} \to \frac{e^{i\alpha}|4\rangle + e^{i\beta}|4'\rangle}{\sqrt{2}} \to \frac{(e^{i\alpha}+e^{i\beta})|5\rangle + (e^{i\alpha}-e^{i\beta})|5'\rangle}{2}$$

$$= (e^{i\alpha}/2)\big[(1+e^{i\Delta})|5\rangle + (1-e^{i\Delta})|5'\rangle\big], \quad \Delta := \beta - \alpha$$

$$\Pr(m=5) = \tfrac{1}{4}|1+e^{i\Delta}|^2 = \tfrac{1}{2}(1+\cos\Delta)$$

$$\Pr(m=5') = \tfrac{1}{4}|1-e^{i\Delta}|^2 = \tfrac{1}{2}(1-\cos\Delta)$$

Toy Measurement

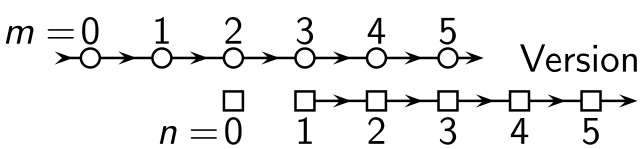


Version I. No time marker.

$T|m,n\rangle = RS|m,n\rangle, \quad S|m,n\rangle = |m+1,n\rangle$

$R|m,n\rangle = |m,n\rangle$ *except* $R|2,n\rangle = |2,1-n\rangle$

$|0,0\rangle \to |1,0\rangle \to |2,1\rangle \to |3,1\rangle \to |4,1\rangle \to \cdots$

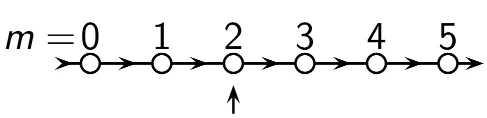


Version II. Time marker.

$T = RS, \quad S|m,n\rangle = |m+1,n+1\rangle$ *except* $S|m,0\rangle = |m+1,0\rangle$

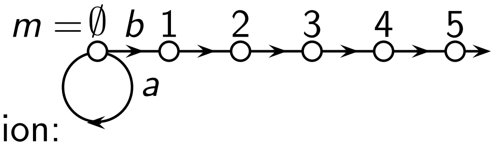
$R|m,n\rangle = |m,n\rangle$ *except* $R|2,0\rangle = |2,1\rangle$

$|0,0\rangle \to |1,0\rangle \to |2,1\rangle \to |3,2\rangle \to |4,3\rangle \to |5,4\rangle \to \cdots$



Version III. Particle as time marker.

Toy Source of Photons



- One photon production:

$|\emptyset\rangle$ = no photon present; $|m\rangle$ = photon at m.

- ∘ Time development

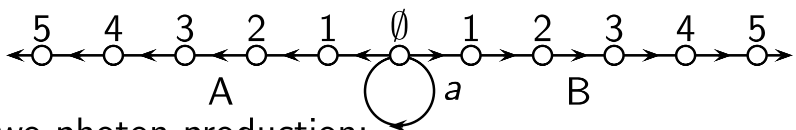
$S|m\rangle = |m+1\rangle$ *except* $S|\emptyset\rangle = a|\emptyset\rangle + b|1\rangle$

$|\emptyset\rangle \;\to\; a|\emptyset\rangle + b|1\rangle \;\to\; a^2|\emptyset\rangle + ab|1\rangle + b|2\rangle$

$\to\; a^3|\emptyset\rangle + a^2b|1\rangle + ab|2\rangle + b|3\rangle \;\to\; \cdots$

- ∘ Assume that $|b| \ll 1$, $|a| \approx 1$, $a = e^{-i\epsilon}$

$|\psi_3\rangle \approx be^{-3i\epsilon}[|\emptyset\rangle + e^{i\epsilon}|1\rangle + e^{2i\epsilon}|2\rangle + e^{3i\epsilon}|3\rangle]$



- Two photon production:

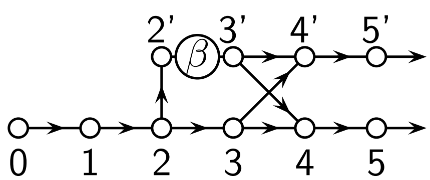
$|m\rangle_A \otimes |n\rangle_B = |m,n\rangle$; $\quad S|m,n\rangle = |m+1,n+1\rangle$ for $m \geq 1,\ n \geq 1$

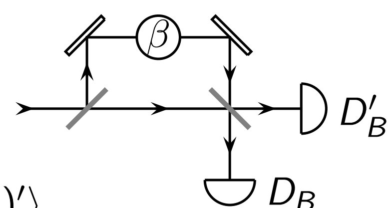
$S|\emptyset\rangle = a|\emptyset\rangle + b|1,1\rangle$

$|\Psi_0\rangle = |\emptyset\rangle \;\;\to\;\; |\Psi_3\rangle \approx$

$be^{-3i\epsilon}[|0,0\rangle + e^{i\epsilon}|1,1\rangle + e^{2i\epsilon}|2,2\rangle + e^{3i\epsilon}|3,3\rangle]$

Unbalanced Mach-Zehnder





$S|m\rangle = |m+1\rangle$, $S|m'\rangle = |(m+1)'\rangle$

$$S|2\rangle = \frac{|3\rangle + |2'\rangle}{\sqrt{2}},\; S|2'\rangle = e^{i\beta}|3'\rangle,\; S|3\rangle = \frac{|4\rangle + |4'\rangle}{\sqrt{2}},\; S|3'\rangle = \frac{|4\rangle - |4'\rangle}{\sqrt{2}}$$

$|0\rangle \to |1\rangle \to |2\rangle \to (1/\sqrt{2})(|2'\rangle + |3\rangle) \to (1/\sqrt{2})e^{i\beta}|3'\rangle + (1/2)(|4\rangle + |4'\rangle)$

$\to \left[e^{i\beta}(|4\rangle - |4'\rangle) + |5\rangle + |5'\rangle\right]/2$ No interference

$|\psi_0\rangle = (|0\rangle + e^{i\epsilon}|1\rangle)/\sqrt{2}; \quad |\psi_5\rangle = S^5|\psi_0\rangle =$

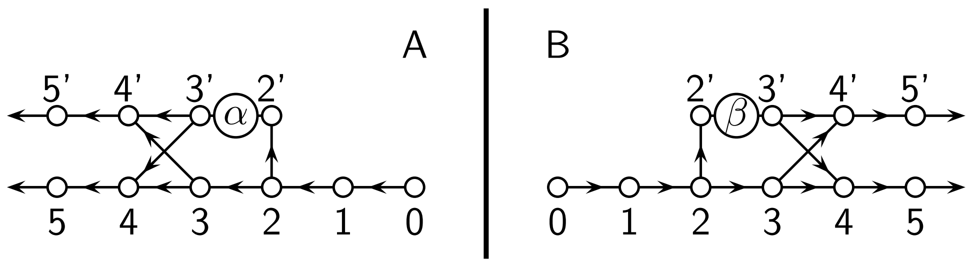
$\left[e^{i\beta}(|4\rangle - |4'\rangle) + (1 + e^{i(\beta+\epsilon)})|5\rangle + (1 - e^{i(\beta+\epsilon)})|5'\rangle + e^{i\epsilon}(|6\rangle + |6'\rangle)\right]/2\sqrt{2}$

Interference in coefficients of $|5\rangle$ and $|5'\rangle$

$\Pr(5) \;= |1 + e^{i(\beta+\epsilon)}|^2/8 = [1 + \cos(\beta + \epsilon)]/4$

$\Pr(5') = |1 - e^{i(\beta+\epsilon)}|^2/8 = [1 - \cos(\beta + \epsilon)]/4$

Toy Franson Interferometer I



- Two photons present simultaneously, so $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

States $|3\rangle_A \otimes |2\rangle_B$ write as $|3,2\rangle$ or $|32\rangle$

Time development $T = S_A \otimes S_B$; S_A same as S_B except α vs β

$$|\Psi_0\rangle = |0,0\rangle = |00\rangle; \quad |\Psi_5\rangle = T^5|\Psi_0\rangle = [S_A^5|0\rangle \otimes S_B^5|0\rangle] = |\psi_5^\alpha\rangle \otimes |\psi_5^\beta\rangle$$

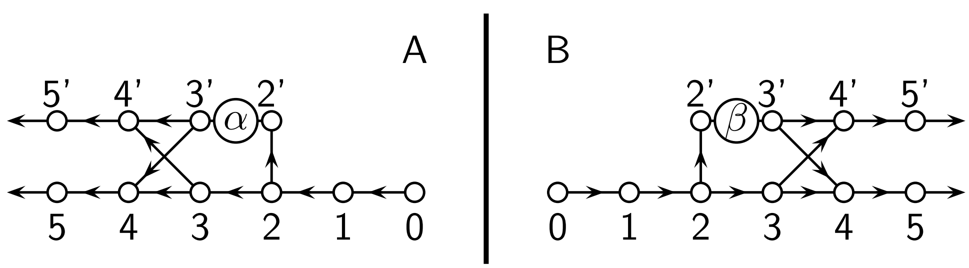
$$= \left[e^{i\alpha}(|4\rangle - |4'\rangle) + |5\rangle + |5'\rangle\right] \otimes \left[e^{i\beta}(|4\rangle - |4'\rangle) + |5\rangle + |5'\rangle\right]$$

$$= \Big\{ e^{i(\alpha+\beta)}[|44\rangle - |44'\rangle - |4'4\rangle + |44\rangle] + e^{i\alpha}[|45\rangle - |4'5\rangle + |45'\rangle - |4'5'\rangle]$$

$$+ e^{i\beta}[|54\rangle - |54'\rangle + |5'4\rangle - |5'4'\rangle] + [|55\rangle + |55'\rangle + |5'5\rangle + |5'5'\rangle]\Big\}/4$$

- Is there interference? Are coincidences special?

Toy Franson Interferometer II



$$|\Phi_0\rangle = |\Psi_0\rangle + e^{i\epsilon}|\Psi_1\rangle = |00\rangle + e^{i\epsilon}|11\rangle; \quad T^5|\Phi_0\rangle = |\Phi_5\rangle = |\Psi_5\rangle + e^{i\epsilon}|\Psi_6\rangle$$

$$= \big\{ e^{i(\alpha+\beta)}[|44\rangle - |44'\rangle - |4'4\rangle + |44\rangle] + [|55\rangle + |55'\rangle + |5'5\rangle + |5'5'\rangle] +$$

$$e^{i(\alpha+\beta+\epsilon)}[|55\rangle - |55'\rangle - |5'5\rangle + |5'5'\rangle] + e^{i\epsilon}[|66\rangle + |66'\rangle + |6'6\rangle + |6'6'\rangle]$$

$$+ \text{ non-coincidence terms } \big\}$$

- Interference terms in the 55, $55'$ coincidences:

$$[1 + e^{i(\alpha+\beta+\epsilon)}]\big(|55\rangle + |5'5'\rangle\big) + [1 - e^{i(\alpha+\beta+\epsilon)}]\big(|55'\rangle + |5'5\rangle\big)$$

$$\Pr(55) = \Pr(5'5') \propto 1+\cos(\alpha+\beta+\epsilon); \quad \Pr(55') = \Pr(5'5) \propto 1-\cos(\alpha+\beta+\epsilon)$$