
FINAL TERM PAPER:

Term papers of length rougly 20 to 25 double-spaced pages (5000 to 6000 words, 40,000 to 50,000 characters) are due on Monday, May 12, the last day of final exams. Because grades for graduating students are needed on May 15, your instructors would be very grateful if you were to turn it in earlier!

In writing a term paper you should list the sources from which you obtain material. Express things as much as possible in your own words, for that means you have understood it. If for some reason you want to quote a source directly, indicate that by an endnote or a footnote. Articles on arXiv illustrate common ways of citing material with a bibliography at the end, though other styles are possible. For additional recommendations on quoting material see the following (recommended by the CMU Eberly Center):

http://writingcenter.waldenu.edu/613.htm

See http://www.cmu.edu/academic-integrity/headernav/policies.html for the CMU policy on academic integrity. If in doubt, contact one of the instructors.

READING:

MBQC = "Measurement-Based Quantum Computation" on course web page. The version dated 22 April 2014 has some minor changes from the version posted earlier, in particular the controlled-phase gates use a different symbol. In addition a final Section 6, Universal Graph State, has been added.

QCQI = Nielsen and Chuang, Quantum Computation and Quantum Information

Measurement-based quantum computation: MBQC; M. A. Nielsen, "Cluster-state quantum computation," Reports on Mathematical Physics 57 (2006) 141; arXiv:quant-ph/0504097v2; A. M. Childs, D. W. Leung and M. A. Nielsen, "Unifed deerivations of measurement-based schemes for quantum computation," Phys. Rev. A 71 (2005) 032318; arXiv:quant-ph/0404132.

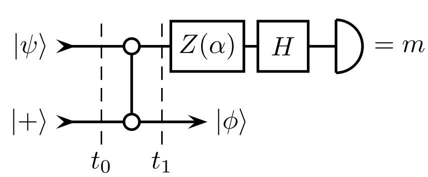
Fault-tolerant quantum computation: QCQI Sec. 10.6; Preskill, Proc. R. Soc. London A **454** 385 (1998), or in *Introduction to Quantum Computation and Information*, edited by H-K Lo, S. Popescu, and T. Spiller (World Scientific, 1998), pp. 213-269. [The two items by Preskill are quite similar, though not identical.]; Gottesman, *An Introduction to Quantum Error Correction and Fault-Tolerant Quantum Computation* arXiv:0904.2557.

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

1b. Only for students enrolled in 33-758: Summarize in half a page to a page what you learned from the most recent seminar.

2. The "fundamental lemma" for measurement-based quantum computation states that for the following circuit



it is the case that

$$|\phi\rangle = HZ^m Z(\alpha)|\psi\rangle,$$

where $m = 0$ or 1 is the outcome of a measurement in the standard basis, and

$$Z(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}, \quad Z = Z(\pi).$$

a) Show that the presence of the one-qubit unitaries preceding the detector in this circuit mean that the measurement is being carried out in an orthonormal basis $\{|a_0\rangle, |a_1\rangle\}$ with $\sqrt{2}|a_0\rangle = |0\rangle + e^{-i\alpha}|1\rangle$ corresponding to $m = 0$ and $|a_1\rangle$ (what is it?) corresponding to $m = 1$.

b) Expand $|\Psi_1\rangle$, the state at t_1 assuming $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$, in the form

$$|\Psi_1\rangle = \sum_j |a_j\rangle \otimes |\beta_j\rangle,$$

and find the $|\beta_j\rangle$.

c) Use the result of (b) to justify the "fundamental lemma." (Remark: the overall phase of $|\phi\rangle$ is unimportant.)

3. The following are linked to Figure 5 in the notes "Measurement-Based Quantum Computation" on the course web page.

a) Suppose that the circuit in part (a) of the figure has an input $|\phi\rangle \otimes |\omega\rangle$ rather than $|+\rangle \otimes |+\rangle$, and the state in part (b) of that figure is that produced by applying CP gates, corresponding to the edges, to a product state in which six of the qubits are in a $|+\rangle$ state, but No. 1 is in the initial state $XZ|\phi\rangle$ —note the Pauli correction compared to what one is trying to do with the circuit in (a)—and No. 5 in the initial state $|\omega\rangle$. (This could arise because of measurements carried out on a larger graph.) Discuss how to carry out the successive measurements on the eight qubits in order to get a result which can be interpreted as the same as if one had used the circuit. Use a notation in which $m_j = 0$ or 1 is the outcome of the measurement on qubit j. Indicate the measurement bases that are required in each case, and how the final measurements on qubits 4 and 8 are to be interpreted.

b) What determines the *order* in which the qubits in the graph state must be measured? In the discussion in the handout it is 1-2-5-6-3-7-4-8. Obviously 5-6-1-2-7-3-8-4 would work equally well. But are there other possibilities? Discuss.

4. a) Use (10.113) in Nielsen and Chuang's discussion of concatenated codes, pp. 480f, to estimate the number of levels of concatenation k required if a circuit involving $N = 10^5$ components ($N = p(n)$ in Nielsen and Chuang) is to perform with an accuracy (probability of failure) of $\epsilon = 10^{-1}$, assuming a threshold of $p_{th} = 1/c = 2 \times 10^{-4}$ and that the probability of failure of a single component is $p = 10^{-4}$. Remember that k must be an integer. If $d = 7$ (corresponding to the Steane code), roughly how many physical components will be present in the simulating circuit?

b) How does your answer to (a) change if you use a more stringent $\epsilon = 10^{-2}$? A less stringent $\epsilon = 0.5$? A larger circuit with $N = 5 \times 10^6$ components and $\epsilon = 0.5$?

c) Your proposal to build a simulating circuit for $N = 5 \times 10^6$ components and $\epsilon = 0.5$ using the k you found in (b) has been turned down by your boss at the FBI because it is too expensive. He proposes building a cheap computer with one less level of concatenation than you have proposed, and running it repeatedly. "After all," he says, "there is a finite probability that it will function properly, so why don't we just keep running it until we manage to break the code?" Calculate the probability that the cheap machine will actually give the right answer on a single run.