

READING MATERIAL:

BORN = “Stochastic Quantum Dynamics I. Born Rule” Course web site
 CQT = Griffiths, Consistent Quantum Theory
 HISTORIES = “Histories and Consistency” Course web site
 MEASURE = “Measurements” Course web site
 PROBS = “Probabilities” Course web site
 QCQI = Nielsen and Chuang, Quantum Computation and Quantum Information
 UNIT = “Unitary Dynamics and Quantum Circuits” Course web site

READING:

Unitary dynamics and quantum circuits: CQT Ch. 7; UNIT;
 QCQI Secs. 1.3.1, 1.3.2, 1.3.4, 1.3.6; 2.2.2; 4.2, 4.3
 Probabilities: PROBS; QCQI App. 1; CQT Ch. 5
 For more extensive discussions of (classical) probability theory: Feller, Vol. 1, or DeGroot and Schervish.
 Born rule: BORN; CQT Secs. 9.2,9.3

READING AHEAD:

Histories and consistency: HISTORIES; CQT Ch. 8; CQT Sec. 11.5
 Measurements: MEASURE; CQT Secs. 7.4; 17.1, 17.2, 18.1, 18.2, 18.3;
 QCQI Secs. 2.2.3, 2.2.4, 2.2.5, 2.2.8, 4.4.

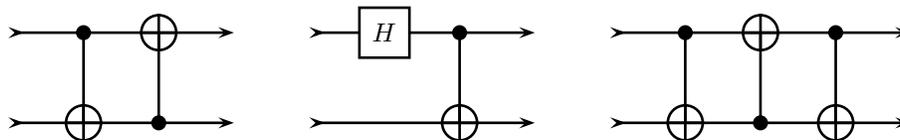
EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.
- 1b. Only for students enrolled in 33-758 (12 units): Summarize in half a page to a page what you learned from the most recent seminar.
2. Any one qubit gate corresponds to a rotation of the Bloch sphere by an angle ω about some axis \mathbf{n} , where $\mathbf{n} = (n_x, n_y, n_z)$ is a unit vector. Find \mathbf{n} and ω for each of the following unitaries:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad T = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}.$$

[Hint. What happens to states such as $|x^+\rangle$ along the coordinate axes?]

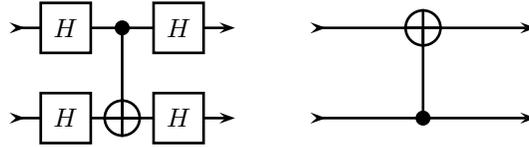
3. Work out the unitary transformations corresponding to the following two-qubit circuits:



Write your answer in two different ways: by giving the action of the unitary transformation on the basis states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$; and as a 4×4 matrix in this same basis. The third circuit is called an “exchange gate”. What does it do to a product state $|a\rangle \otimes |b\rangle$, where $|a\rangle$ and $|b\rangle$ are some linear combinations of $|0\rangle$ and $|1\rangle$?

4. (Do not turn in) Modified form of Nielsen and Chuang Exercise 4.20.

a) Demonstrate the equivalence of the two circuits shown below by working out their action on the standard basis states $|00\rangle$, etc.

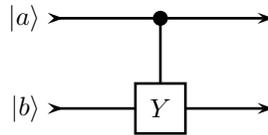


b) Consider a CNOT gate with the first (upper) qubit as the control and the second (lower) qubit the target, and work out what it does to the four states $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$, where for each qubit

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, \quad |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}.$$

Show that if one uses this basis, the second qubit acts as the control, and the first qubit is the target.

5. a) Find the 4×4 unitary matrix corresponding to the following circuit, where Y is the unitary transformation σ_y :



b) The following matrix corresponds to a simple two-qubit circuit involving only one controlled-something gate, where the control could be either the first or the second qubit. Find the circuit.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

6. Experiments on a loaded die show that the probability of $s = 6$ is $1/4$, while $s = 1, 2, 3, 4, 5$ occur with equal probabilities.

a) Find the average value $\langle s \rangle$ of s , and the probability that s is even.

b) The die just discussed is rolled at the same time as an honest die (probability $1/6$ for each s). Describe the sample space. What is the probability that the sum of the two dice is greater than or equal to 10?

c) Same situation as in (b). Find the probability distribution for the six values of s for the loaded die conditional upon the sum of the two being *exactly equal* to 10 (not greater than or equal to 10).

7. Let $T(t_1, t_0)$ be the unitary time development corresponding to a controlled-not gate, with a the control and b the target (data) qubit, and suppose that the initial state is

$$\sqrt{12}|\psi_0\rangle = |a\rangle \otimes |b\rangle = (|0\rangle - i\sqrt{2}|1\rangle) \otimes (\sqrt{3}|0\rangle + |1\rangle)$$

a) Compute the joint probability distribution $\Pr(j, k)$ at t_1 for $|a\rangle = |j\rangle$, $|b\rangle = |k\rangle$, $j, k = 0, 1$, by expanding $|\psi_1\rangle = T(t_1, t_0)|\psi_0\rangle$ in the standard (computational) basis, and taking the absolute squares of the coefficients. Also find the marginal distributions $\Pr(a = j)$ and $\Pr(b = k)$.

b) Find the conditional probability $\Pr(b = k | a = 1)$. Are these variables (one can think of them as S_{az} and S_{bz} , or σ_z^a and σ_z^b) statistically independent?

c) Expand $|\psi_1\rangle$ in the basis $|0+\rangle, |0-\rangle, |1+\rangle, |1-\rangle$, i.e., qubit b in the \pm basis, compute the joint probability distribution $\Pr(a = j, b = \pm)$, and again calculate the marginal distribution $\Pr(a = j)$. You should get the same answer as in (a).

d) Compute the probabilities $\Pr(a = j)$ by expanding $|\psi_1\rangle$ in the form

$$|\psi_1\rangle = |0\rangle \otimes |\beta^0\rangle + |1\rangle \otimes |\beta^1\rangle$$

and computing $\langle \beta^j | \beta^j \rangle$.