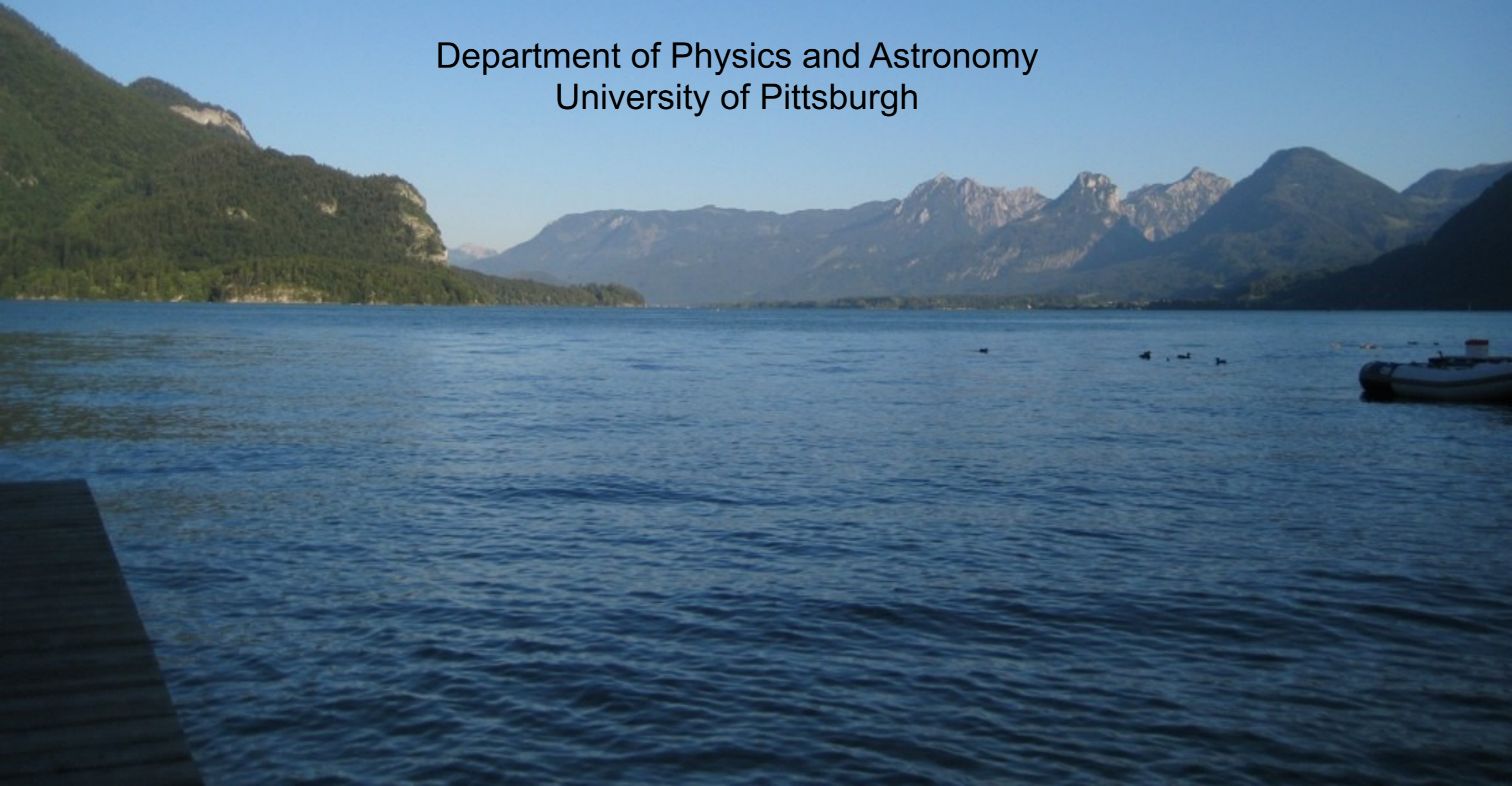


# Physical implementations of quantum computing

Andrew Daley

Department of Physics and Astronomy  
University of Pittsburgh





# Overview

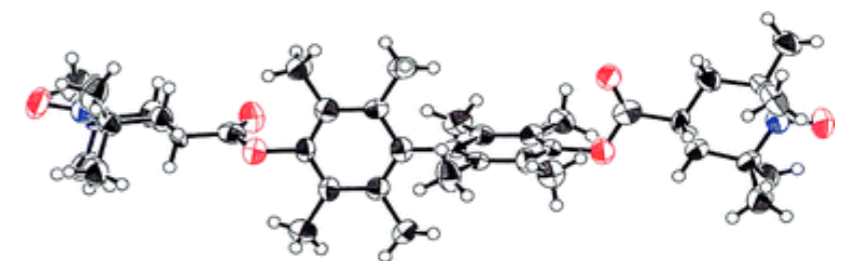
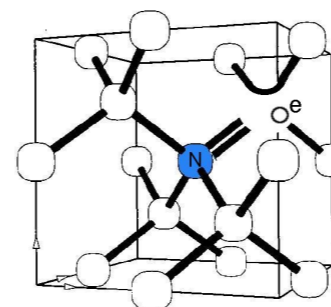
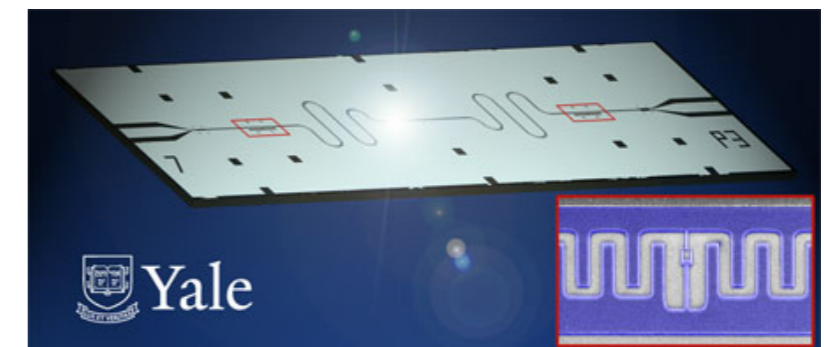
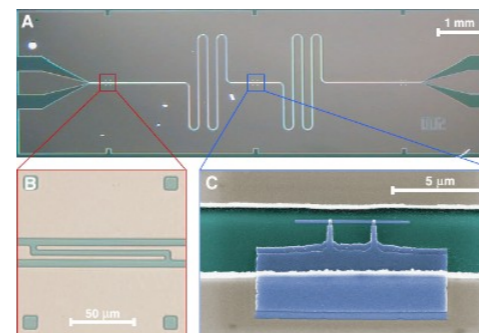
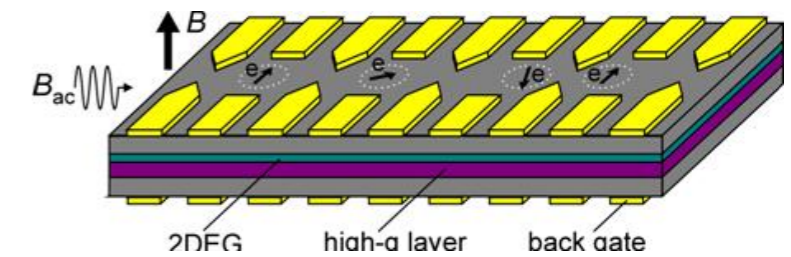
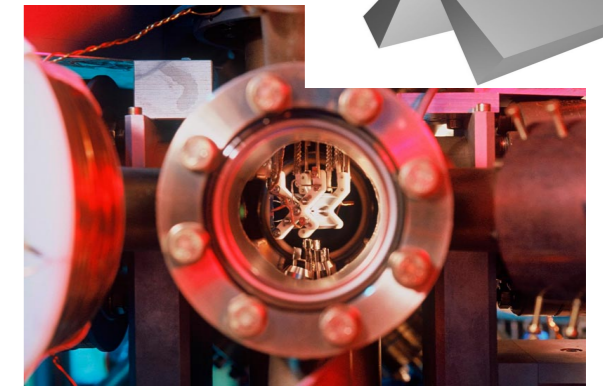
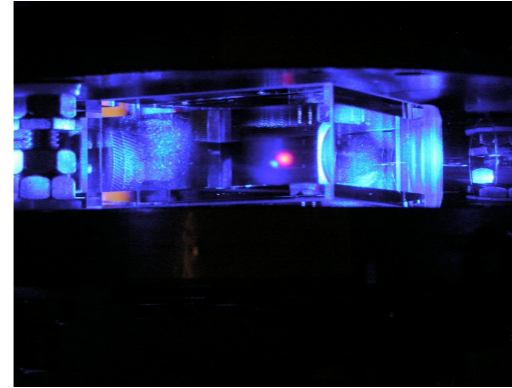
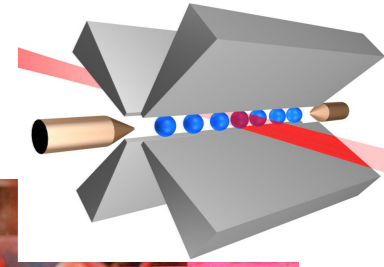
## Introduction

- DiVincenzo Criteria
- Characterising coherence times

## Survey of possible qubits and implementations

- Neutral atoms
- Trapped ions
- Colour centres (e.g., NV-centers in diamond)
- Quantum dots
- Superconducting qubits (charge, phase, flux)
- NMR
- Optical qubits
- Topological qubits

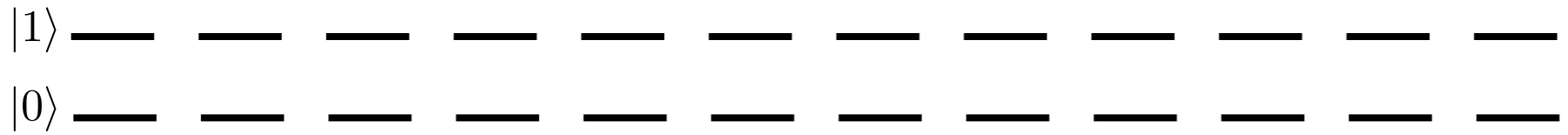
## Summary and comparison



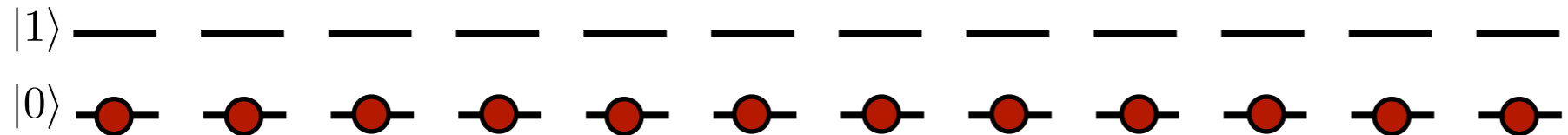
# DiVincenzo Criteria:

## Requirements for the implementation of quantum computation

1. A scalable physical system with well characterized qubits

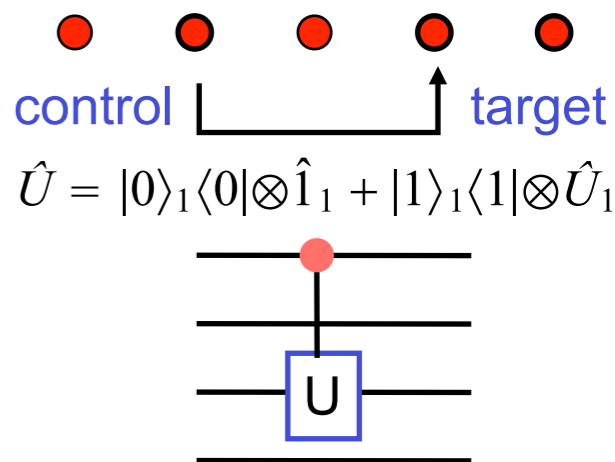
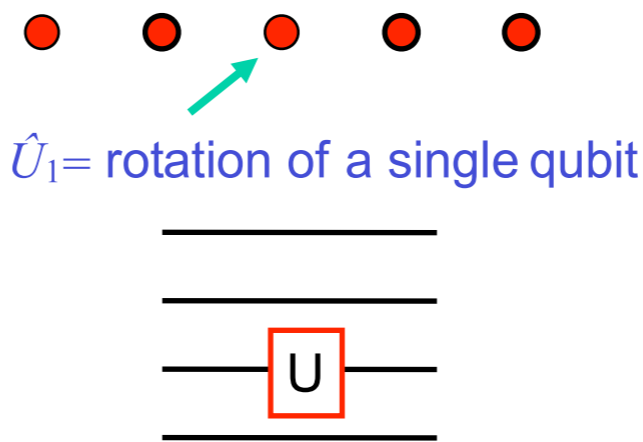


2. The ability to initialize the state of the qubits to a simple fiducial state, such as  $|000\dots\rangle$



3. Long relevant decoherence times, much longer than the gate operation time  
(see next section)

4. A “universal” set of quantum gates  
(single qubit rotations  
+ C-Not / C-Phase / .... )

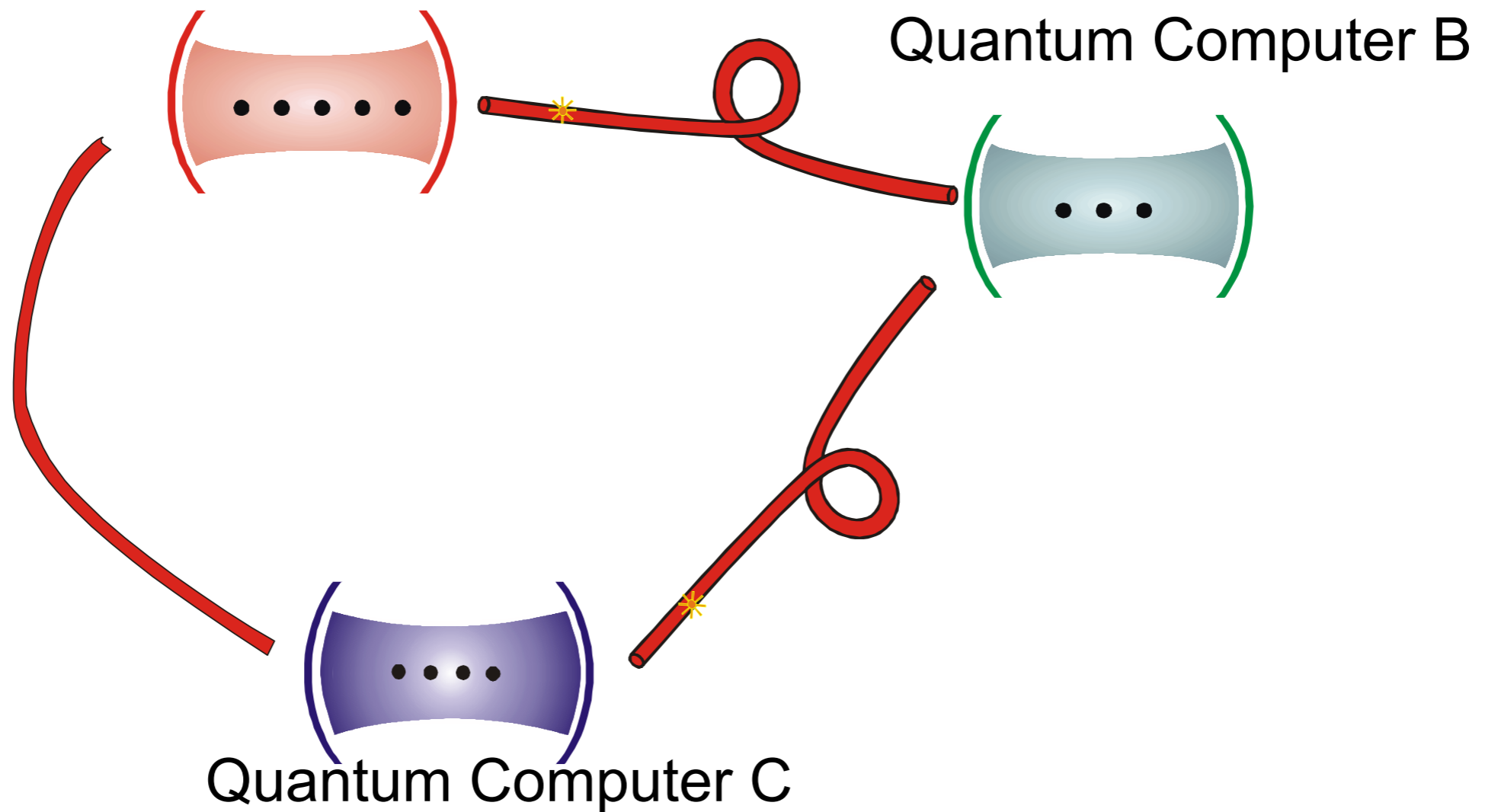


5. A qubit-specific measurement capability

## Desiderata for quantum communication

6. The ability to interconvert stationary and flying qubits
7. The ability faithfully to transmit flying qubits between specified locations

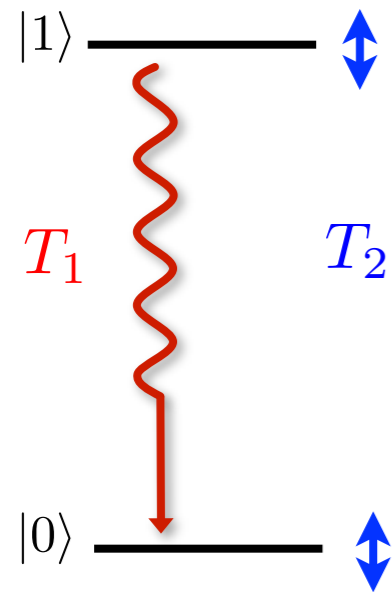
Quantum Computer A





# Characterising coherence times

- Coherence times for qubits are characterized by the timescales:  
(1) for a change in the probability of occupation of either qubit state; and  
(2) for a randomisation of the phase in superposition states



$$\text{State } |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{aligned} \text{Density matrix } \rho &= |\psi\rangle\langle\psi| \\ &\rightarrow \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{pmatrix} \end{aligned}$$

- Timescale  $T_1$  characterises changes in  $|\alpha|^2$  and  $|\beta|^2$
- Timescale  $T_2$  characterises decay of  $\alpha\beta^*$ ,  $\beta\alpha^*$  (loss of purity)
- Typically,  $T_2 < T_1$

- For ensemble measurements (e.g., repeated measurements with fluctuating parameters or multiple qubits in inhomogeneous environments), the system may appear to decohere, due to averaging on a timescale  $T_2^* < T_2$
- This can often be corrected, e.g., by spin-echo experiments that remove the averaging over the fluctuating parameter

**Neutral atoms**

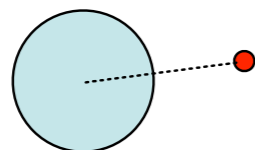
Periodic Table of the Elements

1A 1 <b>H</b> hydrogen 1.008	2A 2 <b>He</b> helium 4.003																																		
3 <b>Li</b> lithium 6.941	4 <b>Be</b> beryllium 9.012	3A 5 <b>B</b> boron 10.81	4A 6 <b>C</b> carbon 12.01	5A 7 <b>N</b> nitrogen 14.01	6A 8 <b>O</b> oxygen 16.00	7A 9 <b>F</b> fluorine 19.00	8A 10 <b>Ne</b> neon 20.18																												
11 <b>Na</b> sodium 22.99	12 <b>Mg</b> magnesium 24.31	13 <b>Al</b> aluminum 26.98	14 <b>Si</b> silicon 28.09	15 <b>P</b> phosphorus 30.97	16 <b>S</b> sulfur 32.07	17 <b>Cl</b> chlorine 35.45	18 <b>Ar</b> argon 39.95	3B 21 <b>Sc</b> scandium 44.96	4B 22 <b>Ti</b> titanium 47.88	5B 23 <b>V</b> vanadium 50.94	6B 24 <b>Cr</b> chromium 52.00	7B 25 <b>Mn</b> manganese 54.94	8B 26 <b>Fe</b> iron 55.85	27 <b>Co</b> cobalt 58.93	28 <b>Ni</b> nickel 58.69	11B 29 <b>Cu</b> copper 63.55	12B 30 <b>Zn</b> zinc 65.39	31 <b>Ga</b> gallium 69.72	32 <b>Ge</b> germanium 72.58	33 <b>As</b> arsenic 74.92	34 <b>Se</b> selenium 78.96	35 <b>Br</b> bromine 79.90	36 <b>Kr</b> krypton 83.80												
37 <b>Rb</b> rubidium 85.47	38 <b>Sr</b> strontium 87.62	39 <b>Y</b> yttrium 88.91	40 <b>Zr</b> zirconium 91.22	41 <b>Nb</b> niobium 92.91	42 <b>Mo</b> molybdenum 95.94	43 <b>Tc</b> technetium (95)	44 <b>Ru</b> ruthenium 101.1	45 <b>Rh</b> rhodium 102.9	46 <b>Pd</b> palladium 106.4	47 <b>Ag</b> silver 107.9	48 <b>Cd</b> cadmium 112.4	49 <b>In</b> indium 114.8	50 <b>Sn</b> tin 118.7	51 <b>Sb</b> antimony 121.8	52 <b>Te</b> tellurium 127.6	53 <b>I</b> iodine 126.9	54 <b>Xe</b> xenon 131.3	55 <b>Cs</b> cesium 132.9	56 <b>Ba</b> barium 137.3	57 <b>La*</b> lanthanum 138.9	72 <b>Hf</b> hafnium 178.5	73 <b>Ta</b> tantalum 180.9	74 <b>W</b> tungsten 183.9	75 <b>Re</b> rhenium 186.2	76 <b>Os</b> osmium 190.2	77 <b>Ir</b> iridium 190.2	78 <b>Pt</b> platinum 195.1	79 <b>Au</b> gold 197.0	80 <b>Hg</b> mercury 200.5	81 <b>Tl</b> thallium 204.4	82 <b>Pb</b> lead 207.2	83 <b>Bi</b> bismuth 208.9	84 <b>Po</b> polonium (209)	85 <b>At</b> astatine (210)	86 <b>Rn</b> radon (222)
87 <b>Fr</b> francium (223)	88 <b>Ra</b> radium (226)	89 <b>Ac~</b> actinium (227)	104 <b>Rf</b> rutherfordium (261)	105 <b>Db</b> dubnium (262)	106 <b>Sg</b> seaborgium (263)	107 <b>Bh</b> bohrium (264)	108 <b>Hs</b> hassium (265)	109 <b>Mt</b> meitnerium (266)	110 <b>Ds</b> darmstadtium (271)	111 <b>Uuu</b> ununundium (272)	112 <b>Uub</b> ununbium (277)	114 <b>Uuq</b> ununquadium (296)		116 <b>Uuh</b> ununhexium (298)		118 <b>Uuo</b> ununoctium (?)																			
Lanthanide Series*		58 <b>Ce</b> cerium 140.1	59 <b>Pr</b> praseodymium 140.9	60 <b>Nd</b> neodymium 144.2	61 <b>Pm</b> promethium (147)	62 <b>Sm</b> samarium 150.4	63 <b>Eu</b> europium 152.0	64 <b>Gd</b> gadolinium 157.3	65 <b>Tb</b> terbium 158.9	66 <b>Dy</b> dysprosium 162.5	67 <b>Ho</b> holmium 164.9	68 <b>Er</b> erbium 167.3	69 <b>Tm</b> thulium 168.9	70 <b>Yb</b> ytterbium 173.0	71 <b>Lu</b> lutetium 175.0																				
Actinide Series~		90 <b>Th</b> thorium 232.0	91 <b>Pa</b> protactinium (231)	92 <b>U</b> uranium (238)	93 <b>Np</b> neptunium (237)	94 <b>Pu</b> plutonium (242)	95 <b>Am</b> americium (243)	96 <b>Cm</b> curium (247)	97 <b>Bk</b> berkelium (247)	98 <b>Cf</b> californium (249)	99 <b>Es</b> einsteinium (254)	100 <b>Fm</b> fermium (253)	101 <b>Md</b> mendelevium (256)	102 <b>No</b> nobelium (254)	103 <b>Lr</b> lawrencium (257)																				



element names in **blue** are liquids at room temperature  
 element names in **red** are gases at room temperature  
 element names in black are solids at room temperature

Rb:



Electron outside closed shell

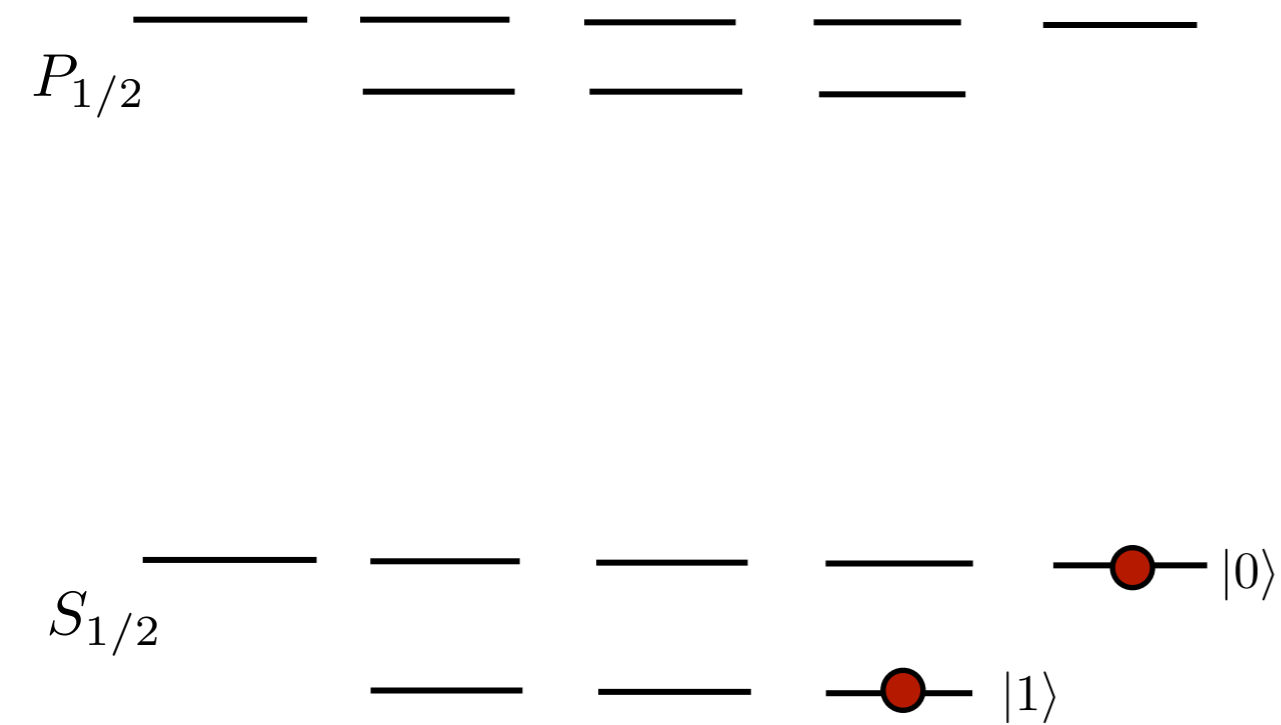
Quantum numbers:  $n, l, m_l$   
 $m_l = -l, \dots, l$

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$  core



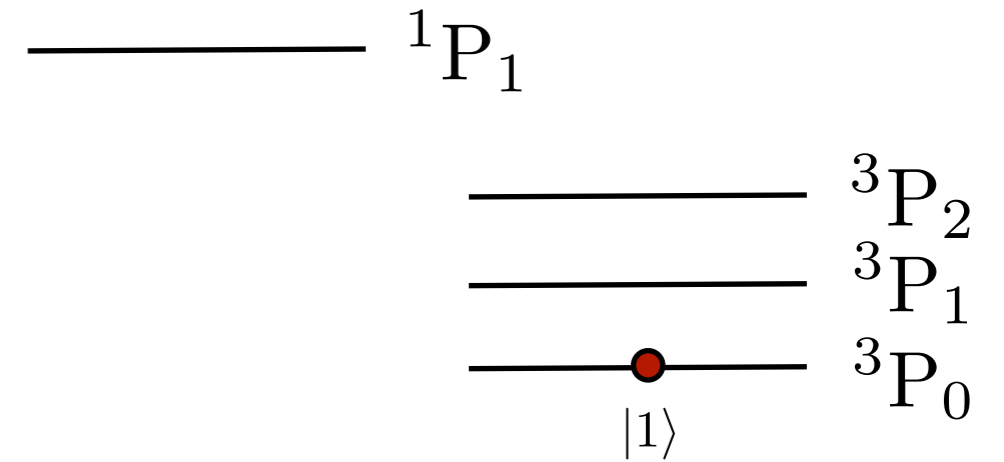
## Alkali atoms

- e.g., Rb, Li, K, Cs,.....
- Qubits encoded on long-lived hyperfine states

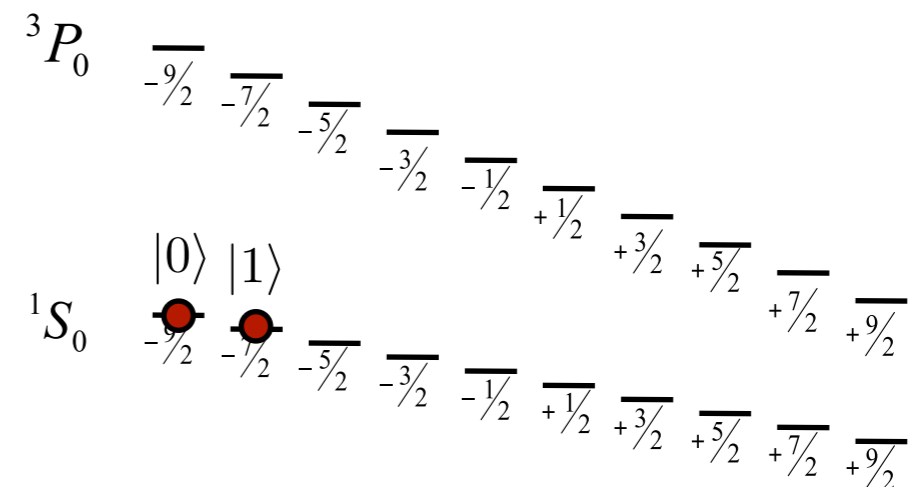


## Alkaline earth (-like) atoms

- e.g., Sr, Yb,.....
- Metastable electronic states



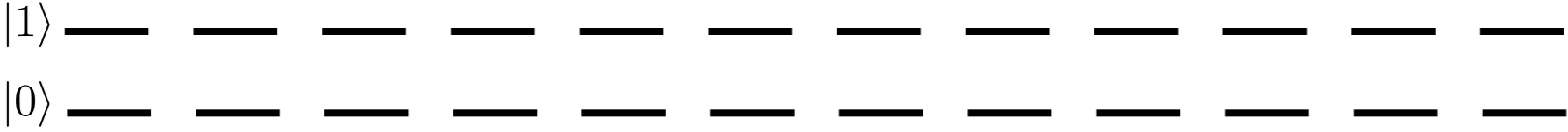
- or nuclear spin states



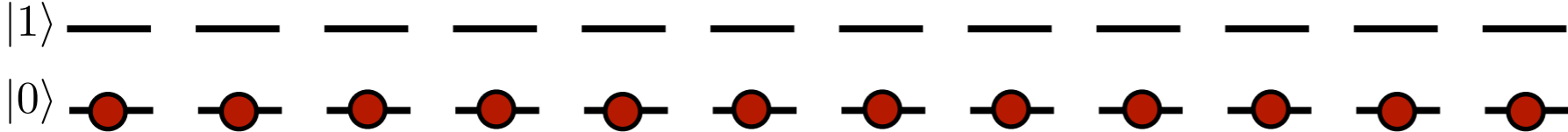
# Back to the DiVincenzo Criteria:

## Requirements for the implementation of quantum computation

1. A scalable physical system with well characterized qubits

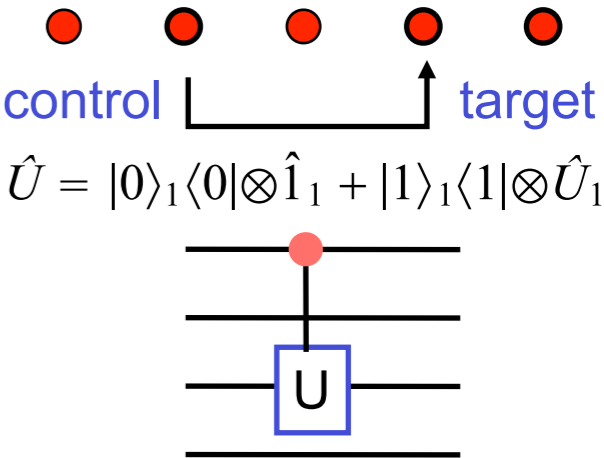
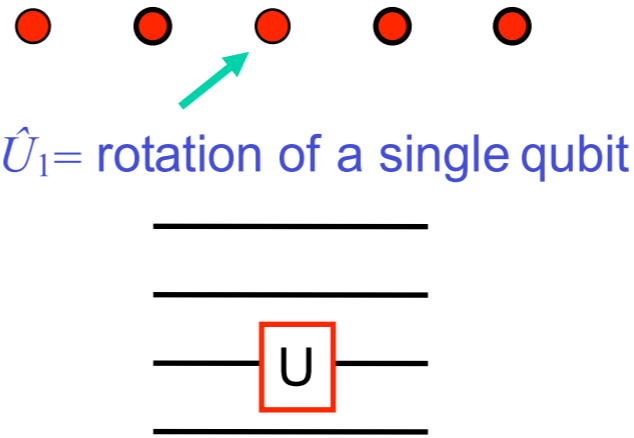


2. The ability to initialize the state of the qubits to a simple fiducial state, such as  $|000\dots\rangle$



3. Long relevant decoherence times, much longer than the gate operation time

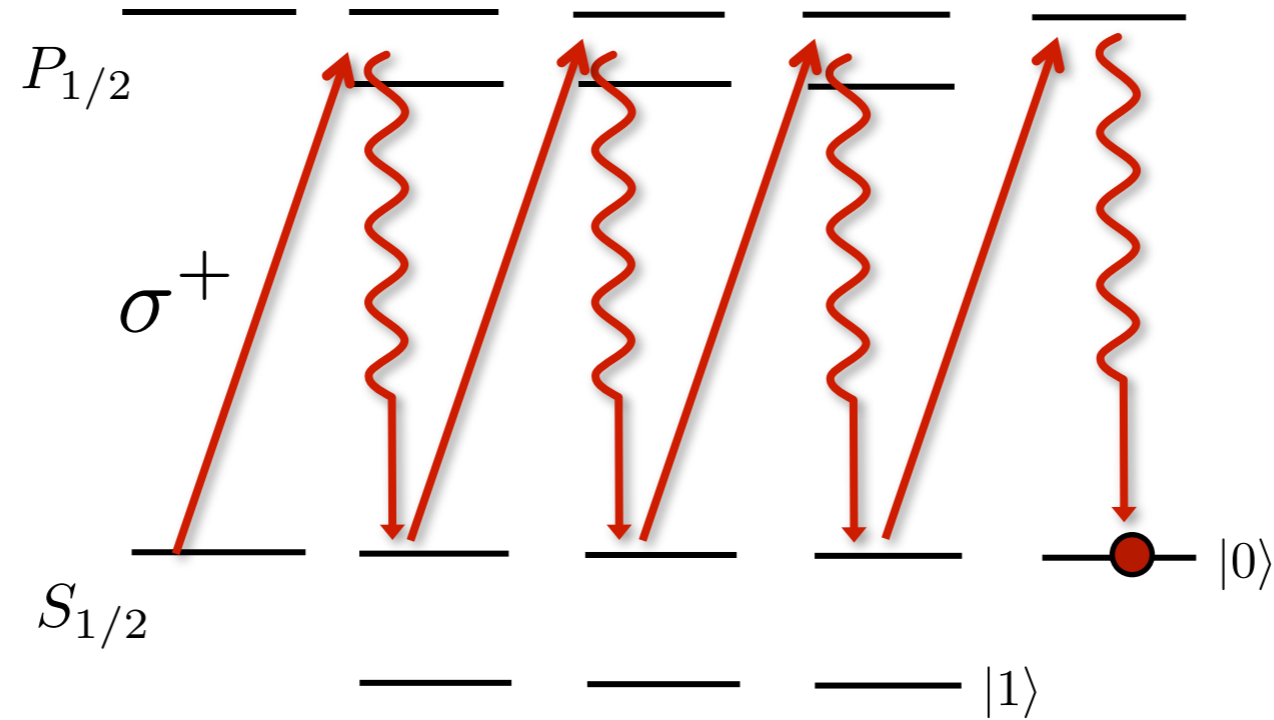
4. A “universal” set of quantum gates  
(single qubit rotations  
+ C-Not / C-Phase / .... )



5. A qubit-specific measurement capability

## Initialisation

- Optical pumping



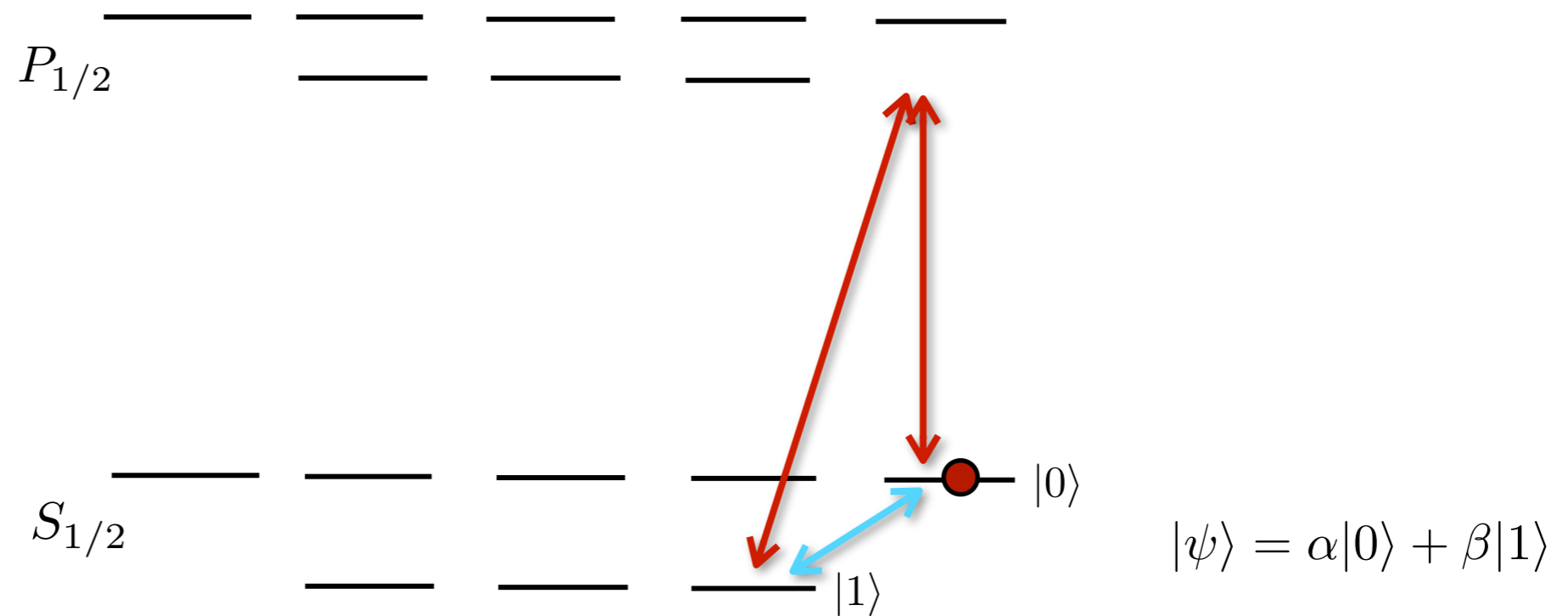
## Decoherence times

- $T_1$  is very long (e.g., generated by blackbody radiation,  $T_1 > 100\text{s}$ )
- $T_2$  is limited, e.g., by inhomogeneous magnetic field fluctuations (measured scales  $\sim 100\text{ms} - 20\text{s}$ )



## Single-qubit gates

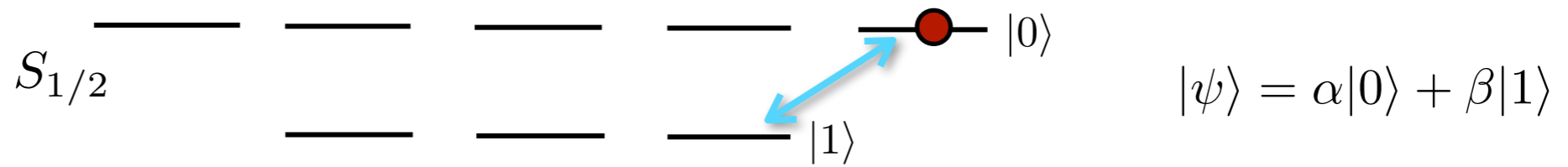
- Raman transitions or radio frequency / microwave fields



- In appropriate parameter regimes, resonant laser coupling will generate the equations of motion (Schrödinger equation)

$$i\frac{d}{dt} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2}\Omega(t) \\ -\frac{1}{2}\Omega(t) & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where we make a Rotating wave approximation, and write the qubit states in the rotating frame. This treatment is excellent (good to within a factor of  $\sim 10^{-8}$  in typical experiments). We have also chosen the phase of  $\Omega$  so that it is real at the location of the qubit.



$$i \frac{d}{dt} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2}\Omega(t) \\ -\frac{1}{2}\Omega(t) & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- Introducing  $\tau = \int_0^t \Omega(t) dt$  as a new time variable we can solve this equation exactly:

$$\begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = U_t \begin{pmatrix} \alpha(0) \\ \beta(0) \end{pmatrix}$$

$$U_t = \begin{pmatrix} \cos \frac{1}{2}\tau & -i \sin \frac{1}{2}\tau \\ -i \sin \frac{1}{2}\tau & \cos \frac{1}{2}\tau \end{pmatrix}$$

- In particular a so-called  $\pi$ -pulse  $\int_{-\infty}^{+\infty} \Omega(t) dt = \pi$  inverts the TLS,

$$U_{t=\pi/\Omega} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \quad \left[ \begin{array}{l} |g\rangle \rightarrow -i|e\rangle \\ |e\rangle \rightarrow -i|g\rangle \end{array} \right]$$

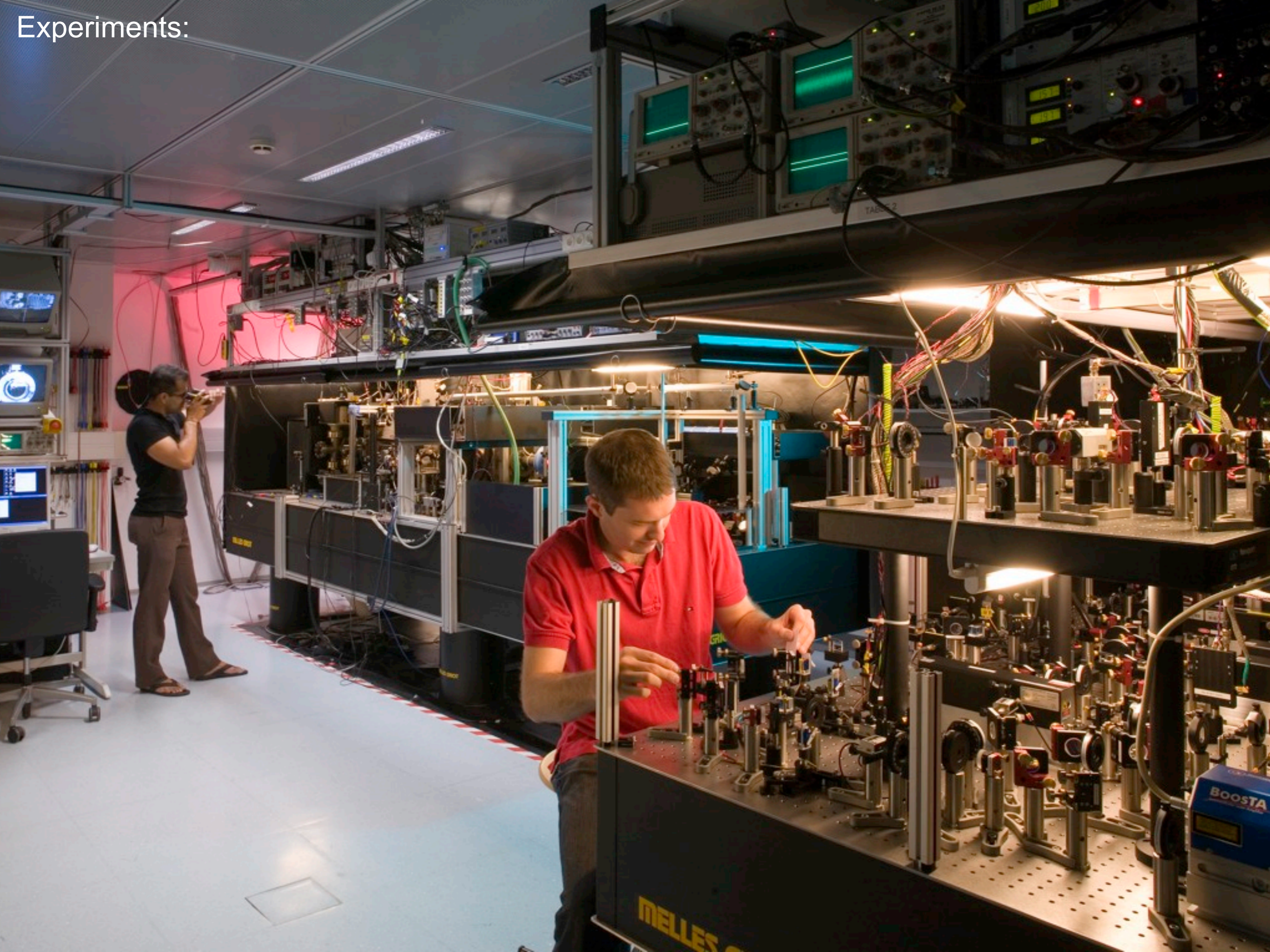
and a  $2\pi$  pulse returns the atom to its ground state,

$$U_{t=2\pi/\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \left[ \begin{array}{l} |g\rangle \rightarrow -|g\rangle \\ |e\rangle \rightarrow -|e\rangle \end{array} \right]$$

but with a negative sign for the amplitudes (rotation of a spin-1/2 by  $2\pi$  changes the sign).

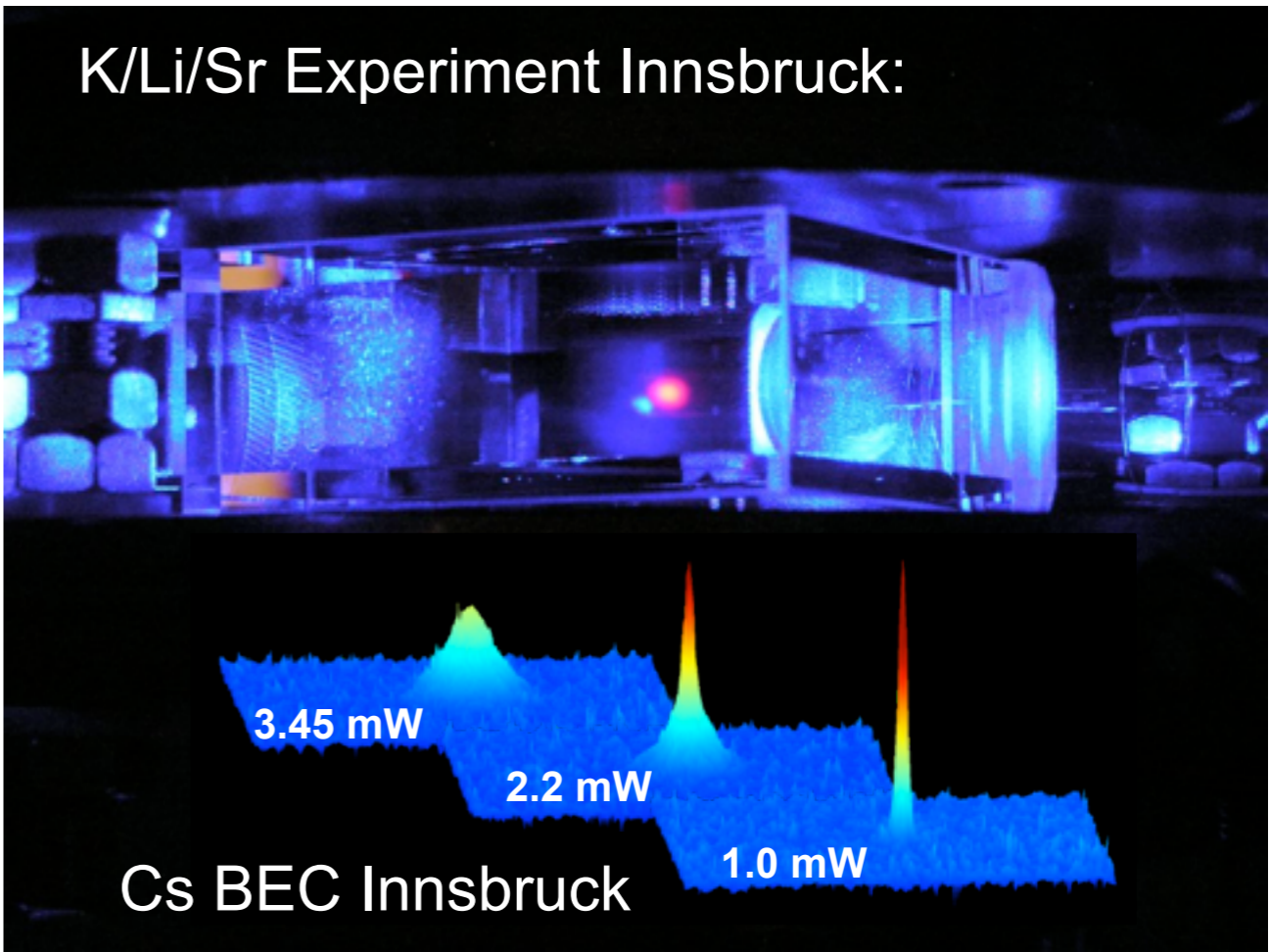
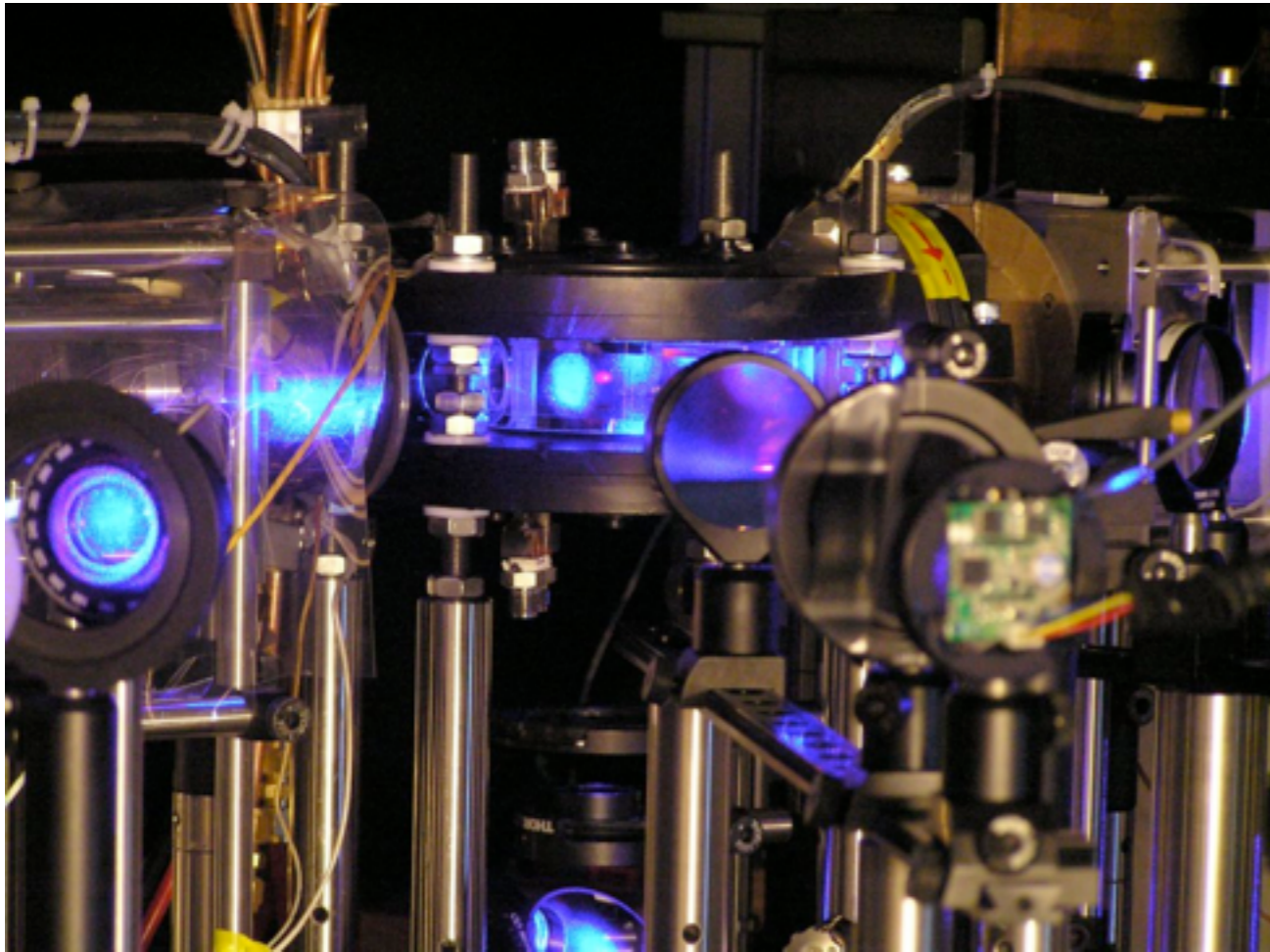


Experiments:



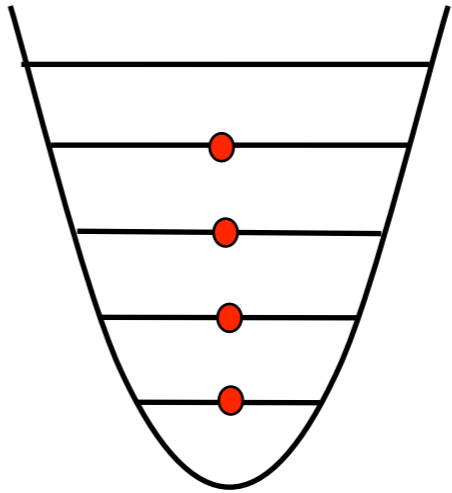


# Degenerate Bose/Fermi Gases in the laboratory:

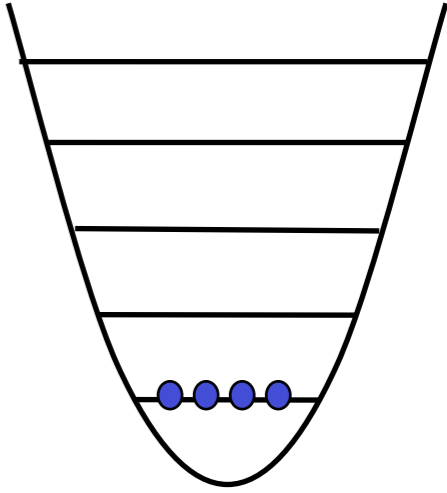


- Features:
  - Control via magnetic field / laser light
  - Microscopically well understood systems

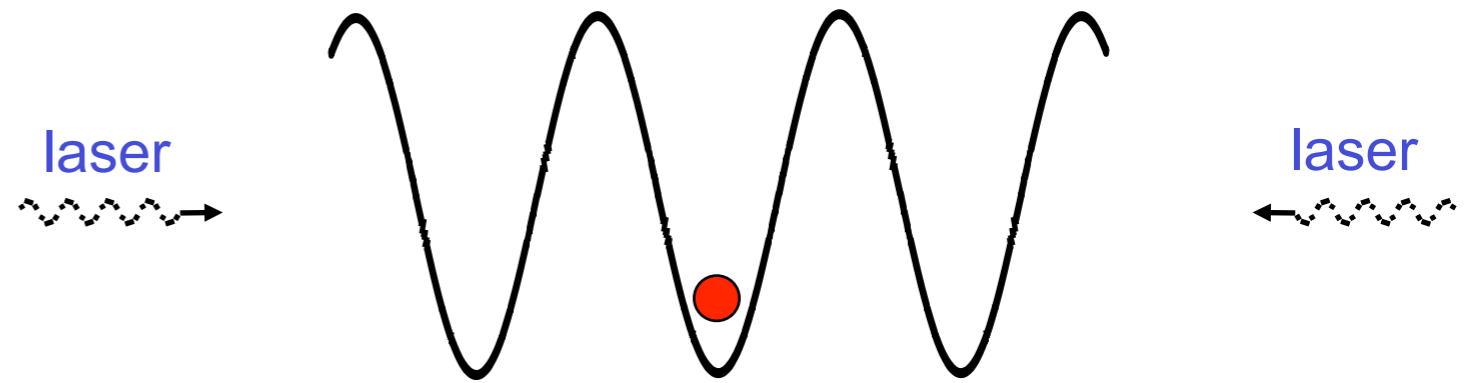
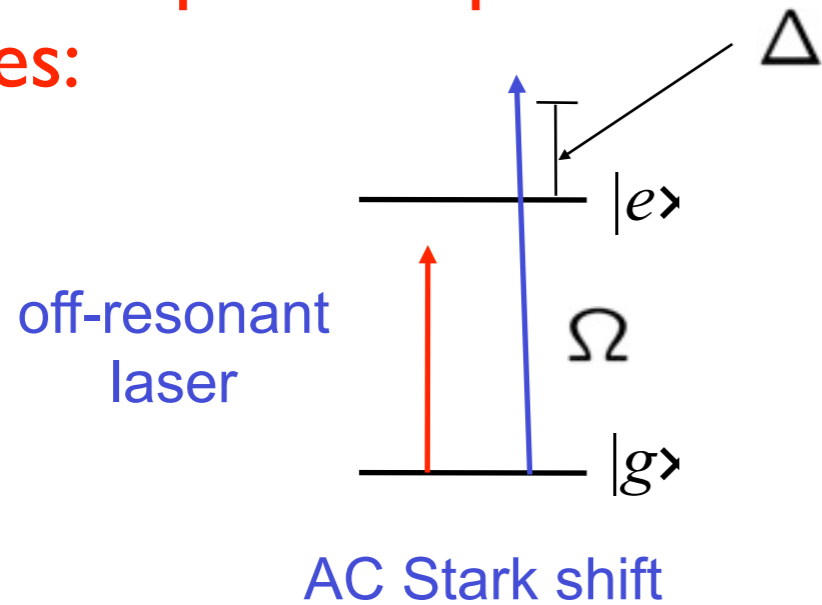
Degenerate Fermi Gas



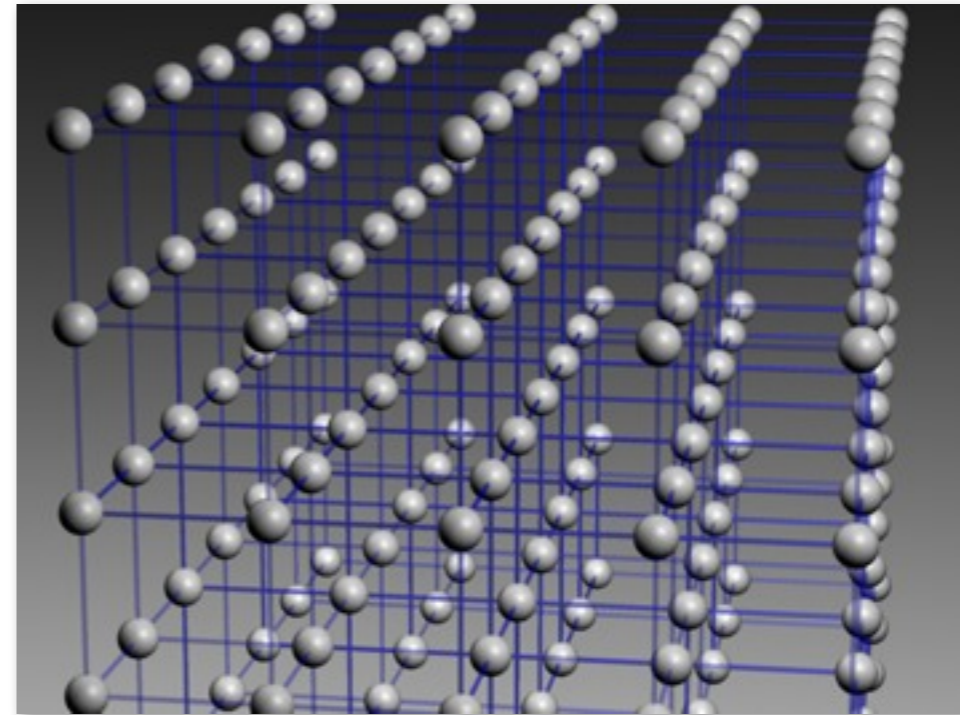
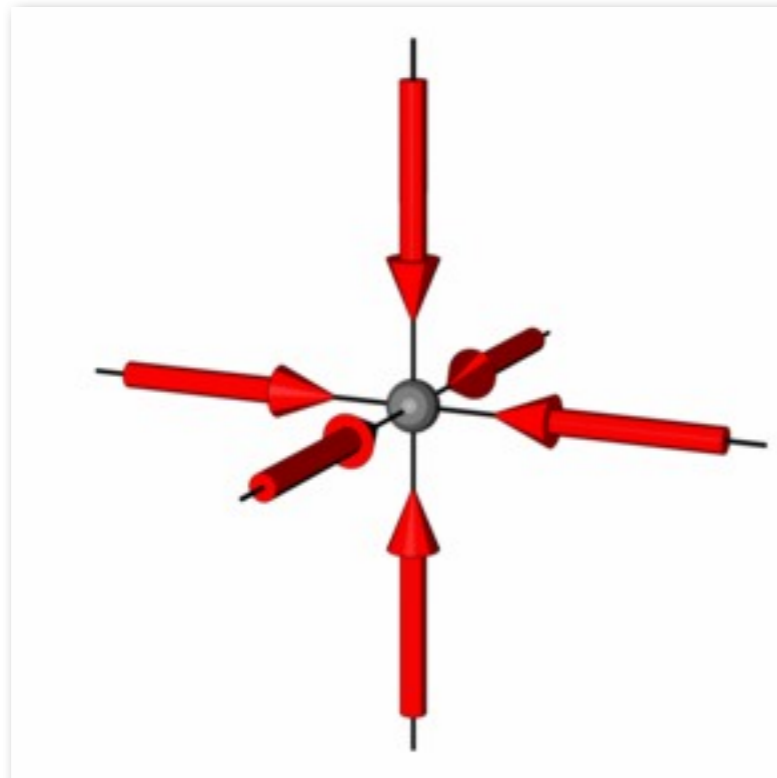
Bose-Einstein Condensate



# Dipole traps and optical lattices:

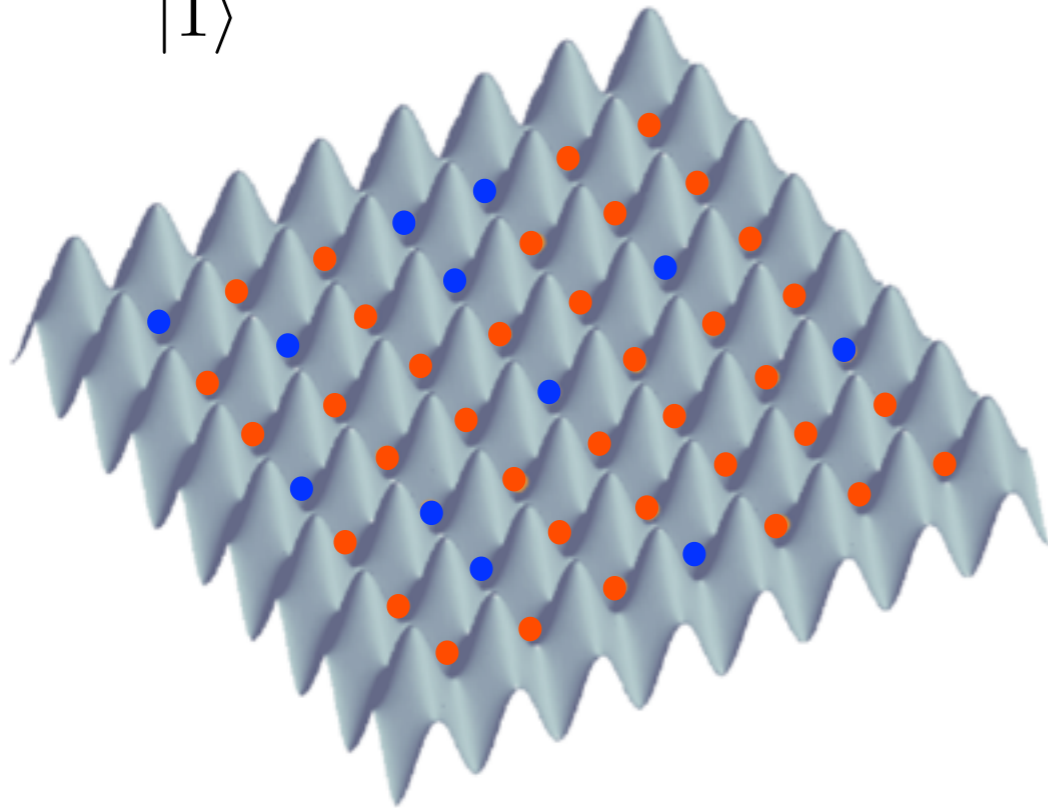
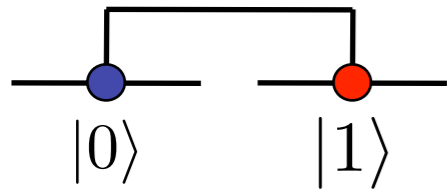


$$V(x) = \alpha(\omega)I = V_0 \sin^2(kx)$$



- Dipole traps for single atoms can be used to trap individual atoms for quantum computing purposes
- Optical lattices allow preparation of a whole register at once, in contrast, e.g., to trapped ions.

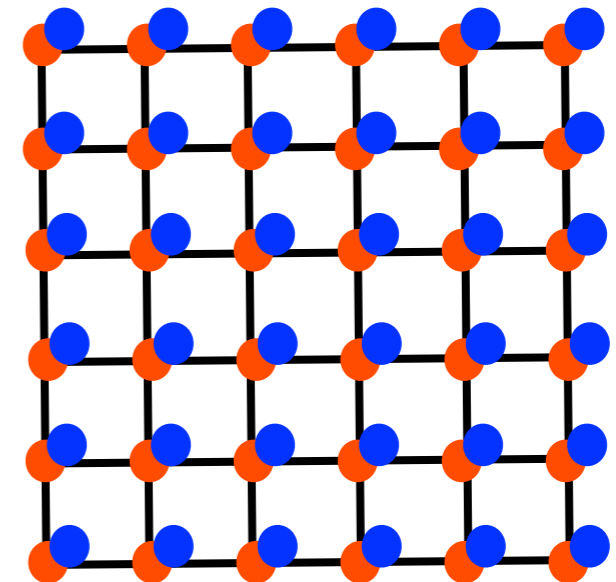
# Quantum Register



## Requirements:

- Long lived storage of qubits
- Addressing of individual qubits
- Single and two-qubit gate operations

- Array of singly occupied sites
- Qubits encoded in long-lived internal states (alkali atoms - electronic states, e.g., hyperfine)
- Entanglement via Rydberg gates or via controlled collisions in a spin-dependent lattice





# Rydberg Blockade Gates:

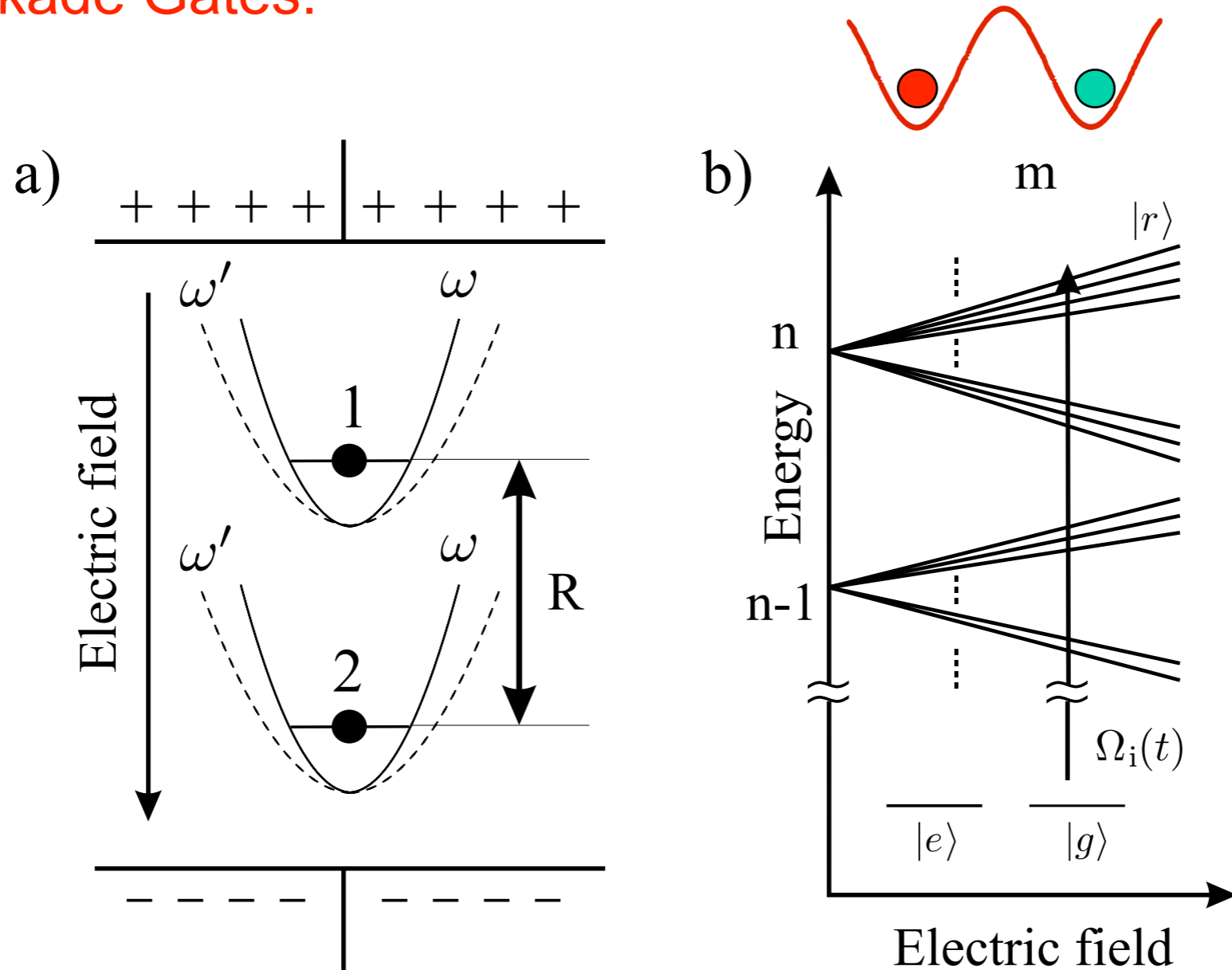
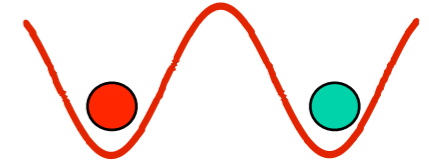


FIG. 1. (a) Setup: A constant electric field along the  $z$  direction is applied to alkali atoms trapped in microtraps. (b) Level scheme: Two ground states  $|g\rangle$  and  $|e\rangle$  (qubits), and laser excitation to the Rydberg state  $|g\rangle \rightarrow |r\rangle$ .

- Excitation of atoms to Rydberg states (highly excited electronic states), with long-range interactions



## Rydberg Blockade Gates:



- Excitation of atom 1 ( $\pi$ -pulse  $g \leftrightarrow r$ )
- Excitation and deexcitation of atom 2 ( $2\pi$ -pulse  $g \leftrightarrow r$ )
- Deexcitation of atom 1 ( $\pi$ -pulse  $g \leftrightarrow r$ )

Initial	Step 1	Step 2	Step 3
$ ee\rangle$	$ ee\rangle$	$ ee\rangle$	$ ee\rangle$
$ ge\rangle$	$-i re\rangle$	$-i re\rangle$	$- ge\rangle$
$ eg\rangle$	$ eg\rangle$	$- eg\rangle$	$- eg\rangle$
$ gg\rangle$	$-i rg\rangle$	$-i rg\rangle$ (not $ rr\rangle$ )	$- gg\rangle$

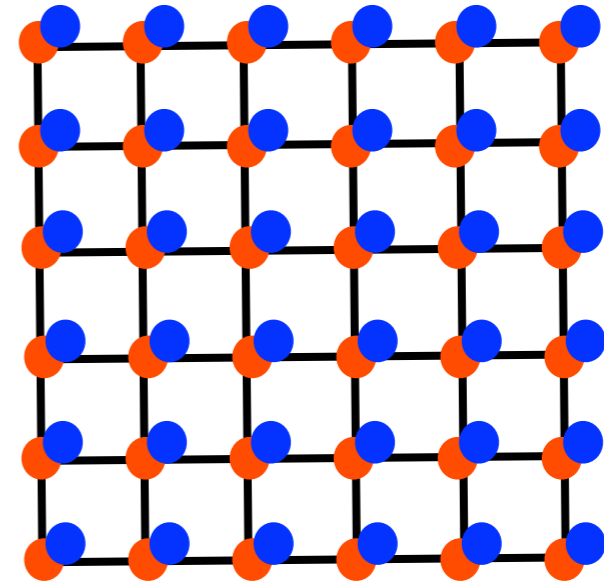
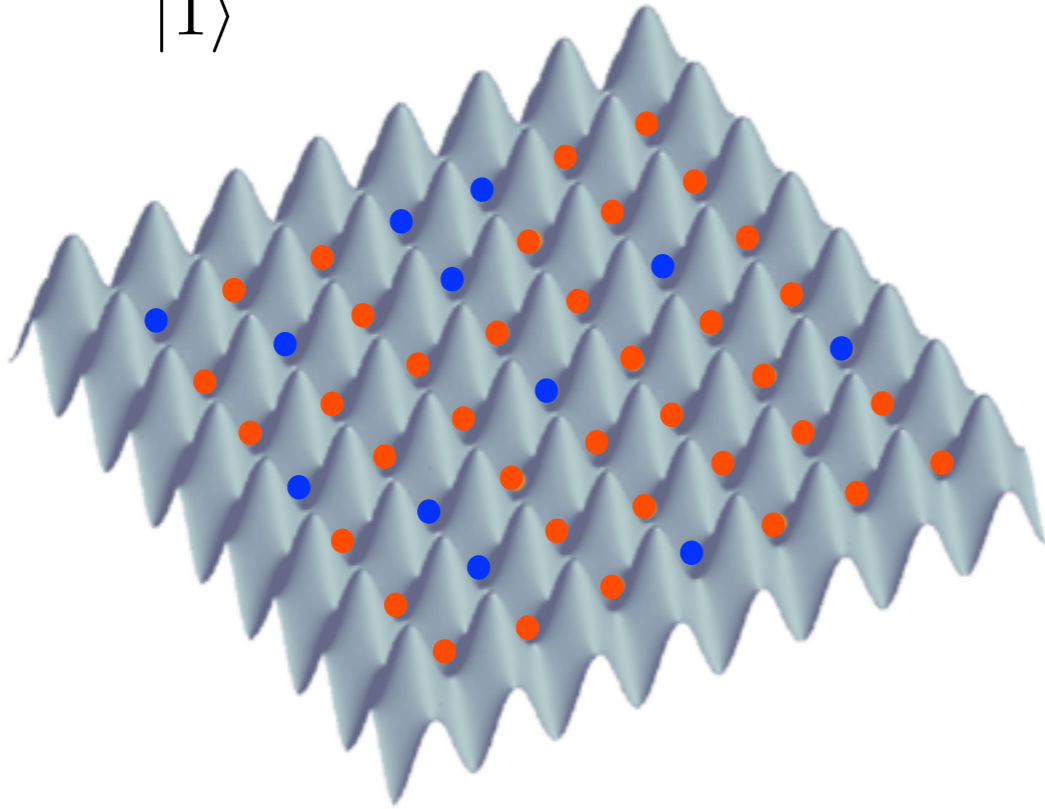
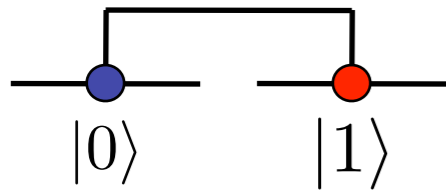
—————  $|ee\rangle$

Jaksch, Cirac, Zoller, Rolston, Coté, Lukin, PRL 85, 2208 (2000)

E. Urban, T.A. Johnson, T. Henage, L. Isenhower, D. D. Yavuz, T. G. Walker, and M. Saffman, Nature Phys. 5, 110 (2009).

A. Gäetan, Y. Miroshnychenko, T. Wilk, A. Chotia, M. Viteau, D. Comparat, P. Pillet, A. Browaeys, and P. Grangier, Nature Phys. 5, 115 (2009).

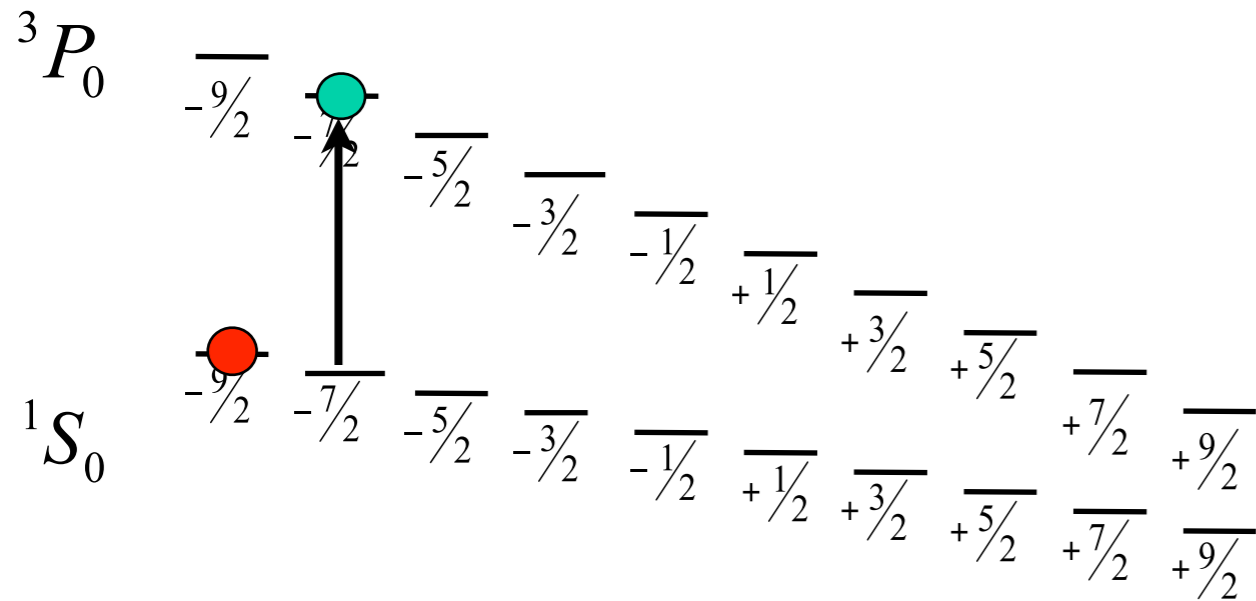
# Gates by controlled collisions



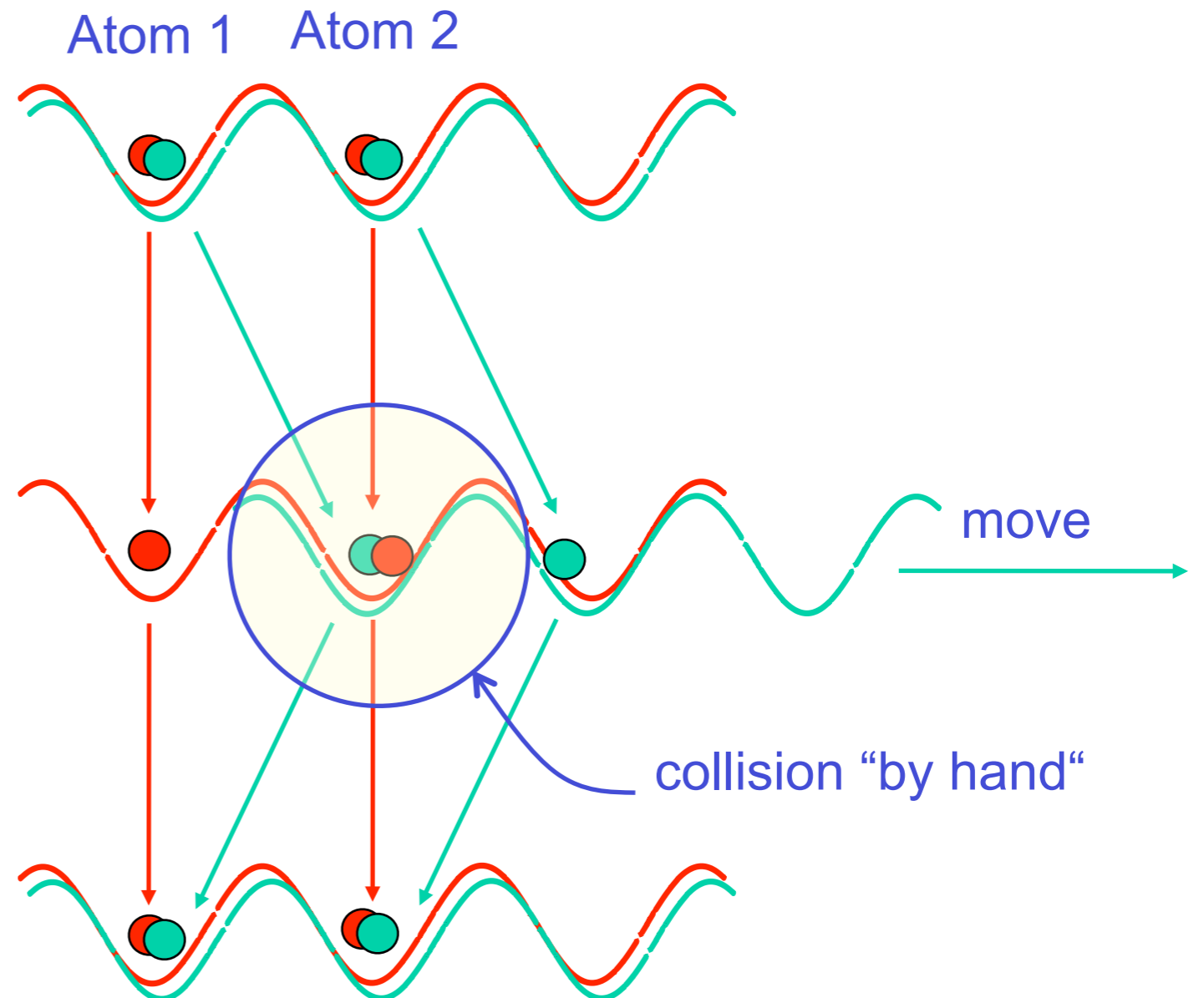
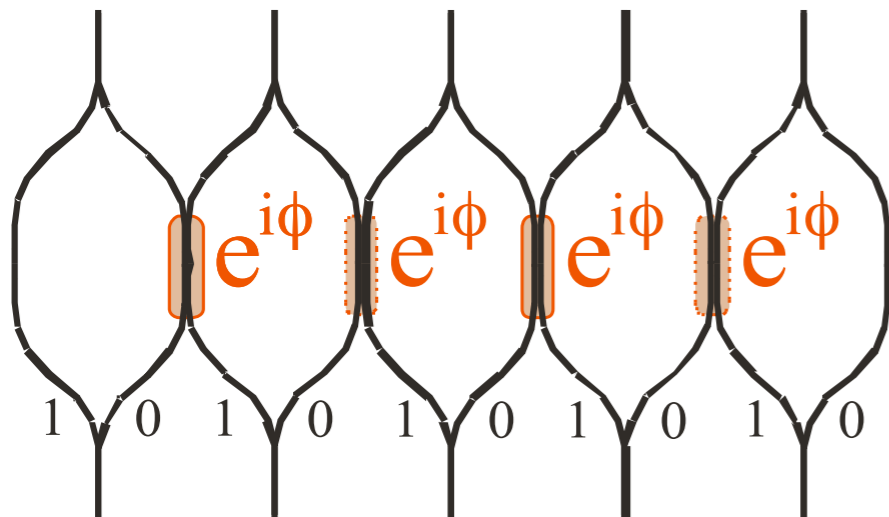
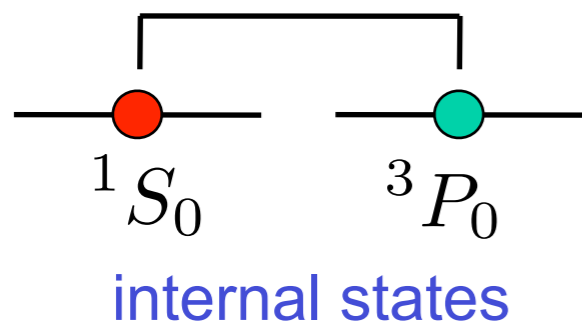
- State - dependent lattice: particles move only if they are in state 0
- If two particles are present on the same site, their energy shifts by  $U$  due to collisions. This leads to a phase accumulation  $e^{-iUT/\hbar}$

Initial	Final
$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$e^{-iUT/\hbar} 01\rangle$
$ 10\rangle$	$ 10\rangle$
$ 11\rangle$	$ 11\rangle$

## Collisional Gates (simple example):



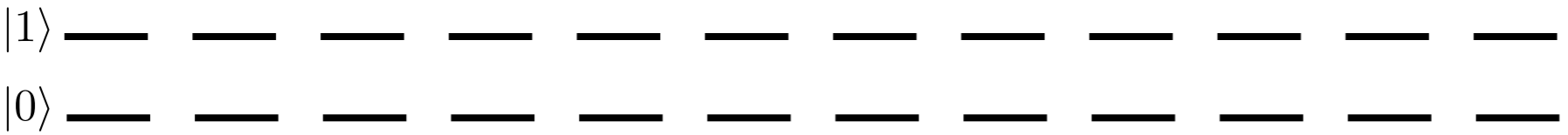
- Gate: controlled collisions  
D. Jaksch et al., PRL 82, 1975 ('99)
- Operation performed in parallel for whole system
- Simple preparation of a cluster state
- Ideal setup for measurement-based quantum computing.



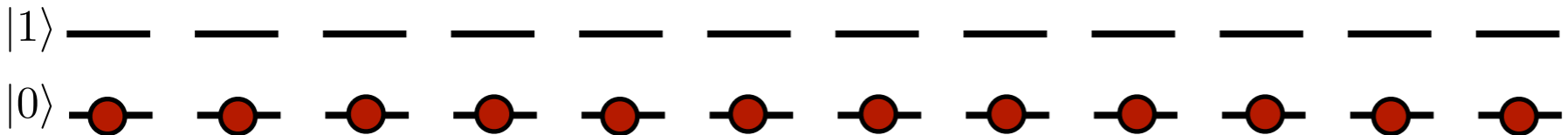
# Back to the DiVincenzo Criteria:

## Requirements for the implementation of quantum computation

1. A scalable physical system with well characterized qubits

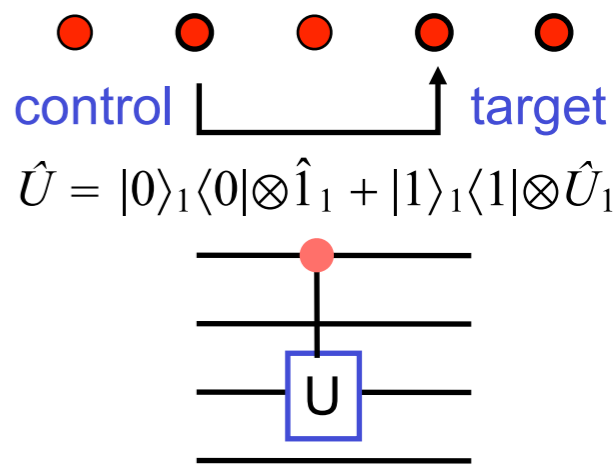
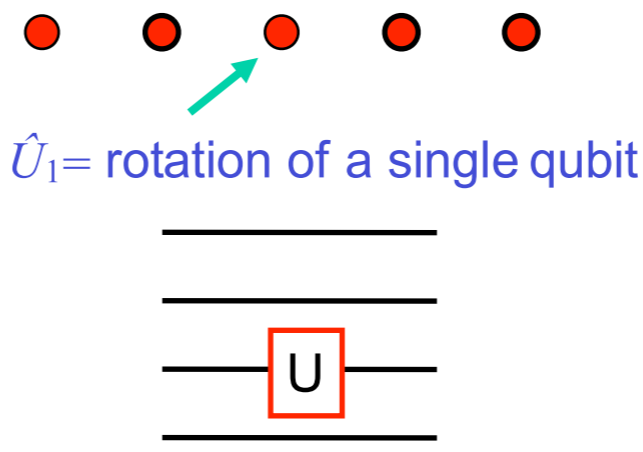


2. The ability to initialize the state of the qubits to a simple fiducial state, such as  $|000\dots\rangle$



3. Long relevant decoherence times, much longer than the gate operation time

4. A “universal” set of quantum gates  
(single qubit rotations  
+ C-Not / C-Phase / .... )

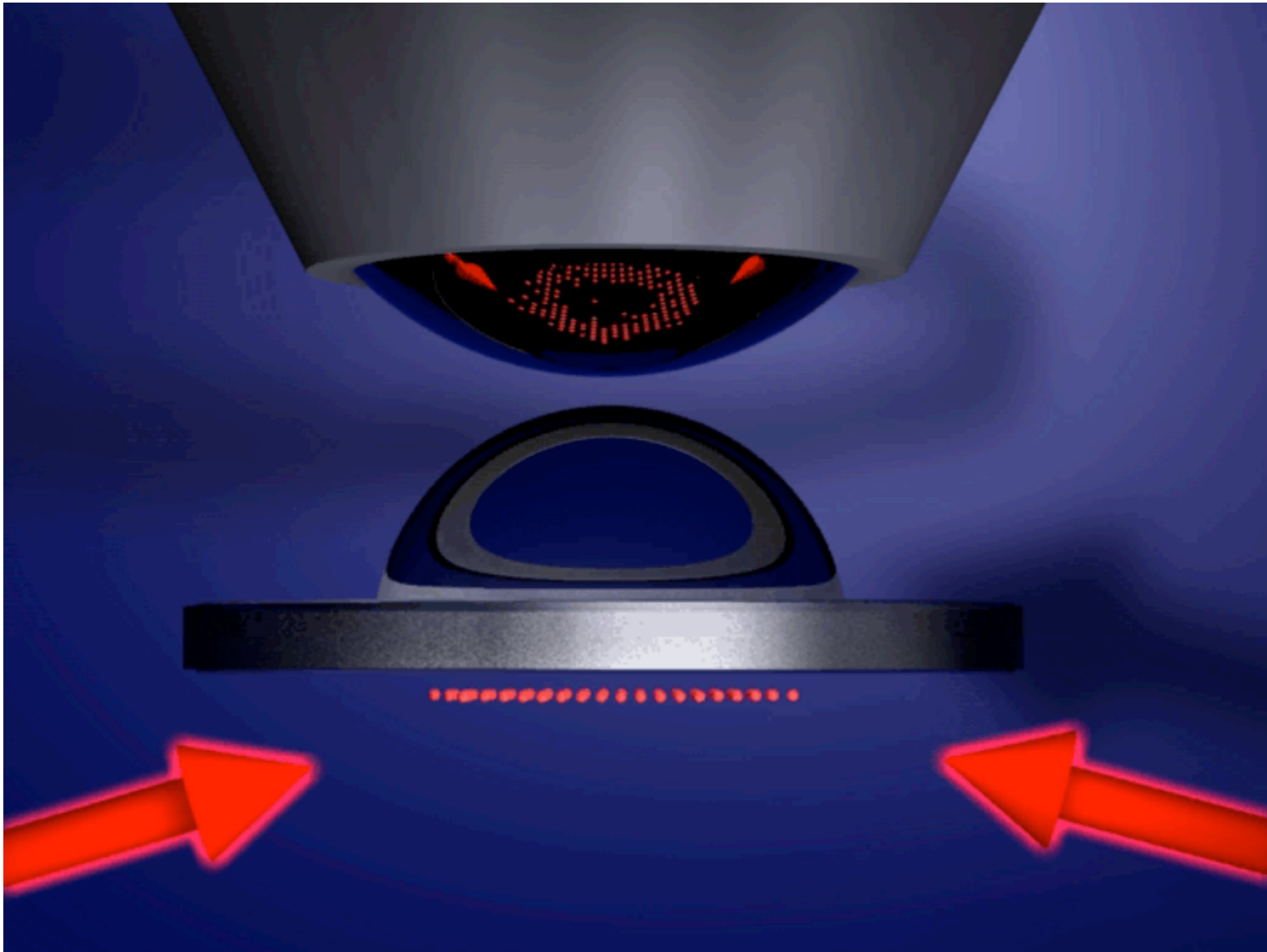


5. A qubit-specific measurement capability

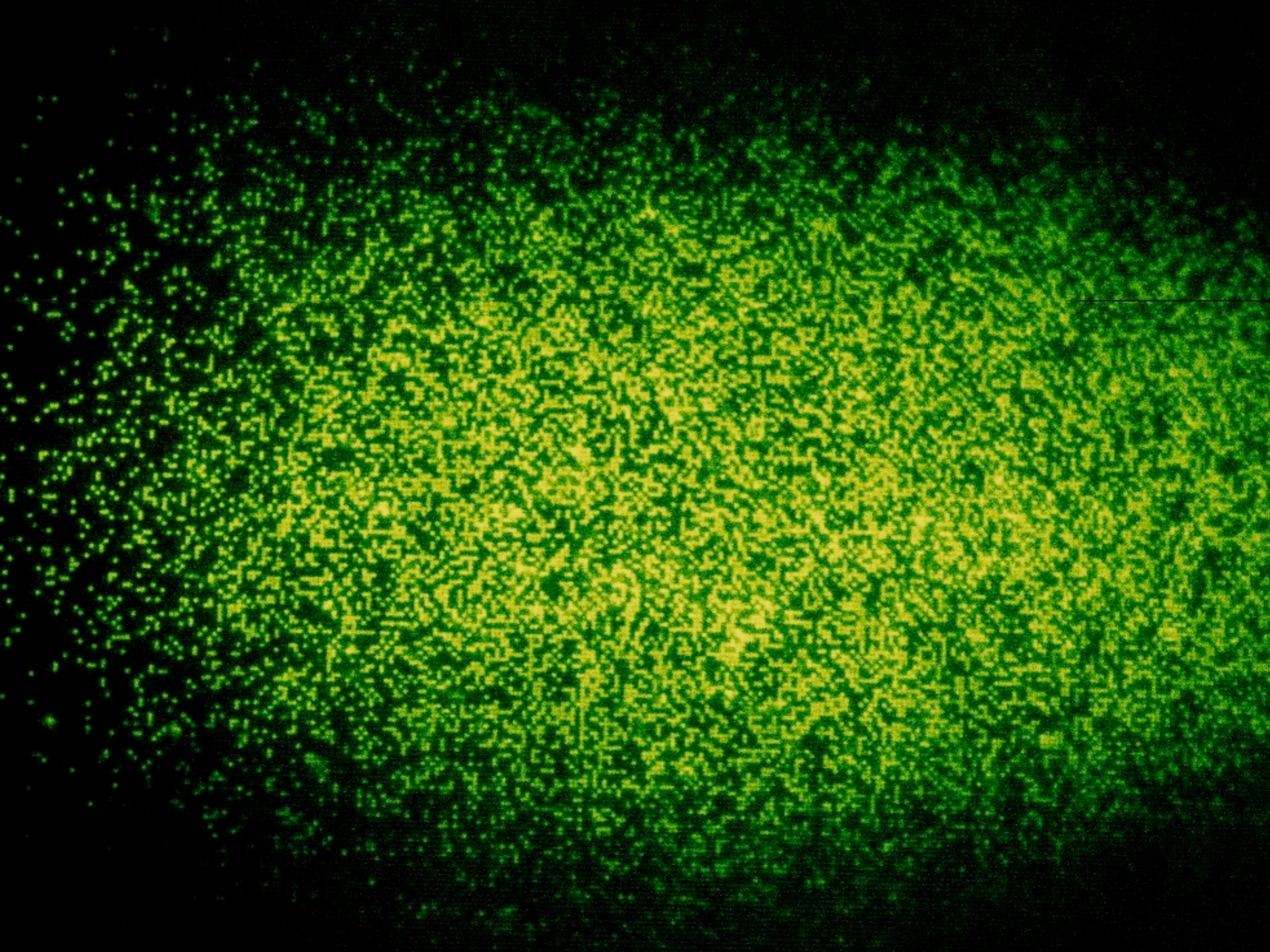


## New Possibility: In-situ measurement of atoms, correlation functions:

- Experiments: Harvard, Chicago, Munich, Oxford, Toronto,.....
- e.g., Markus Greiner's "Microscope" at Harvard:









# New Possibility: In-situ measurement of atoms, correlation functions:

- Experiments: Harvard, Chicago, Munich, Oxford, Toronto,.....
- “Quantum Gas Microscopes” at Harvard / Garching:

W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Foelling, L. Pollet, and M. Greiner, *Science* 329, 547-550 (2010).

C. Weitenberg, M. Endres, J. F. Sherson, M. Cheneau, P. Schauß, T. Fukuhara, I. Bloch, and S. Kuhr, *Nature* 471, 319-324 (2011).

