Chapter 21

Indirect Measurement Paradox

21.1 Statement of the Paradox

The paradox of indirect measurement, often called interaction-free measurement, can be put in a form very similar to the delayed choice paradox discussed in Ch. 20. Consider a Mach-Zehnder interferometer, Fig. 21.1, with two beam splitters, which are always present. A mirror M can be placed either (a) in the c arm of the interferometer, where it reflects the photon out of this arm into channel g, thus preventing it from reaching the second beam splitter, or (b) outside the c arm, in a place where it has no effect. The two positions of M are denoted by M_{in} and M_{out}. Detectors E, F, and G detect the photon when it emerges from the apparatus in channels e, f, or g. With M out of the way, the path differences inside the interferometer are such that a photon which enters through channel a will always emerge in channel f, so the photon will always be detected by F. With M in place, a photon which passes into the c channel cannot reach the second beam splitter B_2. However, a photon which reaches B_2 by passing through the d arm can emerge in either the e or the f channel, with equal probability. As a consequence, for M_{in} the probabilities for detection by E, F, and G are 1/4, 1/4, and 1/2, respectively.

Detection of a photon by G can be thought of as a measurement indicating that the mirror was in the position M_{in} rather than M_{out}. It is a *partial* measurement of the mirror's position in that while a photon detected by G implies the mirror is in place, the converse is not true: the mirror can be in place without the photon being detected by G, since it might have passed through the d arm of the interferometer. Detection of the photon by E can likewise be thought of as a measurement indicating that M is in the c arm, since when M is not there the photon is always detected by F. Detection by E is an indirect measurement that M is in place, in contrast to the direct measurement which occurs when G detects the photon. And detection by E is a also a partial measurement: it can only occur, but does not always occur when M is in the c arm.

The indirect measurement using E seems paradoxical for the following reason. In order to reach E, the photon must have passed through the d arm of the interferometer, since the c arm was blocked by M. Hence the photon was never anywhere near M, and could not have interacted with M. How, then, could the photon have been affected by the presence or absence of the mirror in the c arm, that is, by the difference between M_{in} and M_{out}? How could it "know" that the c arm was blocked, and that therefore it was allowed to emerge (with a certain probability) in the e

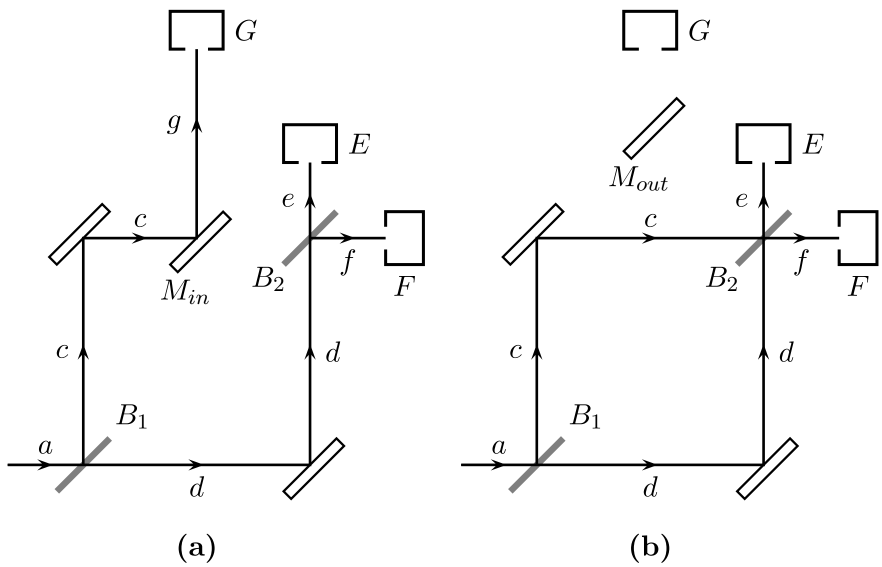

Figure 21.1: Mach-Zehnder interferometer with extra mirror M located (a) in the c arm, (b) outside the interferometer.

channel, an event not possible had M been outside the c arm?

The paradox becomes even more striking in a delayed-choice version analogous to that used in Ch. 20. Suppose the mirror M is initially not in the c arm. However, just before the time of arrival of the photon—that is, the time the photon would arrive were it to pass through the c arm—M is either left outside or rapidly moved into place inside the arm by a servomechanism actuated by a quantum coin flip which took place when the photon had already passed the first beam splitter B_1. In this case one can check, see the analysis in Sec. 21.4 below, that if the photon was later detected in E, M must have been in place blocking the c arm at the instant when the photon would have struck it had the photon been in the c arm. That is, despite the fact that the photon arriving in E was earlier in the d arm it seems to have been sensitive to the state of affairs existing far away in the c arm at just the instant when it would have encountered M! Is there any way to explain this apart from some mysterious nonlocal influence of M upon the photon?

A yet more striking version of the delayed choice version comes from contemplating an extremely large interferometer located somewhere out in space, in which one can arrange that the entire decision process as to whether or not to place M in the c arm occurs during a time when the photon, later detected in E, is at a point on its trajectory through the d arm of the interferometer which is space-like separated (in the sense of relativity theory) from the relevant events in the c arm. Not only does one need non-local influences; in addition, they must travel faster than the speed of light! One way to avoid invoking superluminal signals is by assuming a message is carried, at the speed of light, from M to the second beam splitter B_2 in time to inform the photon arriving in the d arm that it is allowed to leave B_2 in the e channel, rather than having to use the f channel, the only possibility for M_{out}. The problem, of course, is to find a way of getting the message from M to B_2, given that the photon is in the d arm and hence unavailable for this task.

A counterfactual version of this paradox is readily constructed. Suppose that with M in the location M_{in} blocking the c arm, the photon was detected in E. What would have occurred if M_{out} had been the case rather than M_{in}? In particular, if the position of M was decided by a quantum coin toss after the photon was already inside the interferometer, what would have happened to the photon—which must have been in the d arm given that it later was detected by E—if the quantum coin had resulted in M remaining outside the c arm? Would the photon have emerged in the f channel to be detected by F?—this seems the only plausible possibility. But then we are back to asking the same sort of question: how could the photon "know" that the c arm was unblocked?

21.2 Unitary Dynamics

The unitary dynamics for the system shown in Fig. 21.1 is in many respects the same as for the delayed choice paradox of Ch. 20, and we use a similar notation for the unitary time development. Let $|0a\rangle$ be a wave packet for the photon at t_0 in the input channel a just before it enters the interferometer, $|1c\rangle$ and $|1d\rangle$ wave packets in the c and d arms of the interferometer at time t_1, and $|2c\rangle$ and $|2d\rangle$ their counterparts at a time t_2 chosen so that if M is in the c arm the photon will have been been reflected by it into a packet $|2g\rangle$ in the g channel. At time t_3 the photon will have emerged from the second beam splitter in channel e or f—the corresponding wave packets are $|3e\rangle$ and $|3f\rangle$—or will be in a wave packet $|3g\rangle$ in the g channel. Finally, at t_4 the photon will have been detected by one of the three detectors in Fig. 21.1. Their ready states are $|E^\circ\rangle$, $|F^\circ\rangle$ and $|G^\circ\rangle$, with $|E^*\rangle$, $|F^*\rangle$ and $|G^*\rangle$ the corresponding states when a photon has been detected.

The unitary time development from t_0 to t_1 is given by

$$|0a\rangle \mapsto |1\bar{a}\rangle := (|1c\rangle + |1d\rangle)/\sqrt{2}. \tag{21.1}$$

For t_1 to t_2 it depends on the location of M:

$$\begin{aligned} M_{out}:\ |1c\rangle \mapsto |2c\rangle,\ \ |1d\rangle \mapsto |2d\rangle, \\ M_{in}:\ |1c\rangle \mapsto |2g\rangle,\ \ |1d\rangle \mapsto |2d\rangle, \end{aligned} \tag{21.2}$$

with no difference between M_{in} and M_{out} if the photon is in the d arm of the interferometer. For the time step from t_2 to t_3 the relevant unitary transformations are independent of the mirror position:

$$\begin{aligned} &|2c\rangle \mapsto |3\bar{c}\rangle := \big(+|3e\rangle + |3f\rangle\big)/\sqrt{2}, \\ &|2d\rangle \mapsto |3\bar{d}\rangle := \big(-|3e\rangle + |3f\rangle\big)/\sqrt{2}, \\ &|2g\rangle \mapsto |3g\rangle. \end{aligned} \tag{21.3}$$

The detector states remain unchanged from t_0 to t_3, and the detection events between t_3 and t_4 are described by:

$$|3e\rangle|E^\circ\rangle \mapsto |E^*\rangle, \quad |3f\rangle|F^\circ\rangle \mapsto |F^*\rangle, \quad |3g\rangle|G^\circ\rangle \mapsto |G^*\rangle. \tag{21.4}$$

If the photon is not detected, the detector remains in the ready state; thus (21.4) is an abbreviated version of

$$|3e\rangle|E^\circ\rangle|F^\circ\rangle|G^\circ\rangle \mapsto |E^*\rangle|F^\circ\rangle|G^\circ\rangle, \tag{21.5}$$

etc. One could also use macro projectors or density matrices for the detectors, see Sec. 17.4, but this would make the analysis more complicated without altering any of the conclusions.

21.3 Comparing M_{in} and M_{out}

In Sec. 20.3 we considered separate consistent families depending upon whether the second beam splitter was in or out. That approach could also be used here, but for the sake of variety we adopt one which is slightly different: a single family with *two* initial states at time t_0, one with the mirror in and one with the mirror out,

$$|\Psi_0\rangle|M_{out}\rangle, \quad |\Psi_0\rangle|M_{in}\rangle, \tag{21.6}$$

each with a positive (non-zero) probability, where

$$|\Psi_0\rangle = |0a\rangle|E^\circ\rangle|F^\circ\rangle|G^\circ\rangle. \tag{21.7}$$

Here the mirror is treated as an inert object, so $|M_{in}\rangle$ and $|M_{out}\rangle$ do not change with time. They do, however, influence the time development of the photon state as indicated in (21.2). Thus unitary time development of the first of the two states in (21.6) leads at time t_4 to

$$|E^\circ\rangle|F^*\rangle|G^\circ\rangle|M_{out}\rangle, \tag{21.8}$$

while the second results in a macroscopic quantum superposition (MQS) state

$$\tfrac{1}{2}\Big(-|E^*\rangle|F^\circ\rangle|G^\circ\rangle + |E^\circ\rangle|F^*\rangle|G^\circ\rangle + \sqrt{2}|E^\circ\rangle|F^\circ\rangle|G^*\rangle\Big)|M_{in}\rangle. \tag{21.9}$$

Consider the consistent family with support given by the four histories

$$\begin{array}{l} \Psi_0 M_{out} \odot [1\bar{a}] \odot [2\bar{a}] \odot [3f] \odot F^*, \\ \Psi_0 M_{in} \ \odot [1\bar{a}] \odot [2s] \odot [3s] \odot \{E^*, F^*, G^*\}, \end{array} \tag{21.10}$$

at the times $t_0 < t_1 < t_2 < t_3 < t_4$, where

$$|2s\rangle := \big(|2d\rangle + |2g\rangle\big)/\sqrt{2}, \quad |3s\rangle := \tfrac{1}{2}\big(-|3e\rangle + |3f\rangle + \sqrt{2}|3g\rangle\big) \tag{21.11}$$

are superposition states in which the photon is not located in a definite channel. This corresponds to unitary time development until the photon is detected, and then pointer states (as defined at the end of Sec. 9.5) for the detectors. It shows that E^* and G^* can only occur with M_{in}, and in this sense either of these events constitutes a measurement indicating that the mirror was in the c arm. There is nothing paradoxical about the histories in (21.10), because an important piece of the paradox stated in Sec. 21.1 was the notion that the photon detected by E must at an earlier time have been in the d arm of the interferometer. But since the projectors C and D for the particle to be in the c or the d arm do not commute with $[1\bar{a}]$, the assertion that the photon was in one or the other arm of the interferometer at time t_1 makes no sense when one uses (21.10), and the same is true at t_2.

Therefore let us consider a different consistent family with support

$$\begin{array}{l} \Psi_0 M_{out} \odot \quad [1\bar{a}] \odot [2\bar{a}] \odot \quad [3f] \odot F^*, \\ \Psi_0 M_{in} \odot \begin{cases} [1c] \odot [2g] \odot \quad [3g] \odot G^*, \\ [1d] \odot [2d] \odot \begin{cases} [3e] \odot E^*, \\ [3f] \odot F^*. \end{cases} \end{cases} \end{array} \tag{21.12}$$

Using this family allows us to assert that if the photon was later detected by E, then the mirror was in the c arm, and the photon itself was in the d arm while inside the interferometer, and thus far away from M. The photon states at time t_1 in this family are contextual in the sense discussed in Ch. 14, since $[1c]$ and $[1d]$ do not commute with $[1\bar{a}]$, and the same is true for $[2d]$ at t_2. Thus $[1d]$ and $[2d]$ depend, in the sense of Sec. 14.1, on M_{in}, and it makes no sense to talk about whether the photon is in the c or the d arm if the mirror is out of the way, M_{out}. For this reason it is not possible to use (21.12) in order to investigate what effect replacing M_{in} with M_{out} has on the photon while it is in arm d. Hence while (21.12) represents some advance over (21.10) in stating the paradox, it cannot be used to infer the existence of nonlocal effects.

As noted in Ch. 14, the fact that certain events are contextual should not be thought of as something arising from a physical cause; in particular, it is misleading to think of contextual events as "caused" by the events on which they depend, in the technical sense defined in Sec. 14.1. Thinking that the change from M_{out} to M_{in} in (21.12) somehow "collapses" the photon from a superposition into one localized in one of the arms is quite misleading. Instead, the appearance of a superposition in the M_{out} case and not in the M_{in} case reflects our decision to base a quantum description upon (21.12) rather than, for example, the family (21.10), where the photon is in a superposition both for M_{out} and M_{in}.

One can also use a consistent family in which the photon is in a definite arm while inside the interferometer both when M is in and when it is out of the c arm, so that the c and d states are not contextual:

$$\begin{aligned} \Psi_0 M_{out} \odot &\begin{cases} [1c] \odot [2c] \odot \quad [3\bar{c}] \odot S^+, \\ [1d] \odot [2d] \odot \quad [3\bar{d}] \odot S^-, \end{cases} \\ \Psi_0 M_{in} \odot &\begin{cases} [1c] \odot [2g] \odot \quad [3g] \odot G^*, \\ [1d] \odot [2d] \odot \begin{cases} [3e] \odot E^*, \\ [3f] \odot F^*. \end{cases} \end{cases} \end{aligned} \tag{21.13}$$

The states S^+ and S^- are the macroscopic quantum superposition (MQS) states of detectors E and F as defined in (20.9) and (20.10). Just as in the case of the family (20.17) in Sec. 20.3, the MQS states in the last two histories in (21.13) cannot be eliminated by replacing them with E^* and F^*, as that would violate the consistency conditions. And since the projectors S^+ and S^- do not commute with E^* and F^*, contextuality has not really disappeared when (21.12) is replaced by (21.13): it has been removed from the events at t_1 and t_2, but reappears in the events at t_3 and t_4. In particular, it would make no sense to look at the events at the final time t_4 in (21.13) and conclude that a detection of the photon by E^* was evidence that the mirror M was in rather than out of the c arm. While such a conclusion would be valid using (21.10) or (21.12), it is not supported by (21.13) since in the latter E^* only makes sense in the case M_{in}, and is meaningless with M_{out}.

The preceding analysis has uncovered a very basic problem. Using E^* as a way of determining that M_{in} is the case rather than M_{out} is incompatible with using E^* as an indication that the photon was earlier in the d rather than the c arm. For the former, (21.10) is perfectly adequate, as is (21.12). However, when we try and construct a family in which $[1c]$ vs. $[1d]$ makes sense whether or not the mirror is blocking the c arm, the result, (21.13), is unsatisfactory, both because of the appearance of MQS states at t_4, and also because E^* is now contextual in a way which makes it depend on M_{in}, so that it is meaningless in the case M_{out}. Hence the detection of the photon by E

cannot be used to distinguish M_{in} from M_{out} if one uses (21.13). If this were a problem in classical physics, one could try combining results from (21.10), (21.12), and (21.13) in order to complete the argument leading to the paradox. But these are incompatible quantum frameworks, so the single framework rule means that the results obtained using one of them cannot be combined with results obtained using the others. From this perspective the paradox stated in Sec. 21.1 arises from using rules of reasoning which work quite well in classical physics, but do not always function properly when imported into the quantum domain.

21.4 Delayed Choice Version

In order to construct a delayed-choice version of the paradox, we suppose that a quantum coin is connected to a servomechanism, and during the time interval between t_1 and $t_{1.5}$, while the photon is inside the interferometer but before it reaches the mirror M, the coin is tossed and the outcome fed to the servomechanism. The servomechanism then places the mirror M in the c arm or leaves it outside, as determined by the outcome of the quantum coin toss. Using the abbreviated notation at the end of Sec. 19.2, the corresponding unitary time development from t_1 to $t_{1.5}$ can be written in the form

$$|M_0\rangle \mapsto (|M_{in}\rangle + |M_{out}\rangle)/\sqrt{2}, \tag{21.14}$$

the counterpart of (20.18) for the delayed choice paradox of Ch. 20. Here $|M_0\rangle$ is the initial state of the quantum coin, servomechanism, and mirror. The kets $|M_{in}\rangle$ and $|M_{out}\rangle$ in (21.14) include the mirror and the rest of the apparatus (coin and servomechanism), and thus they have a slightly different physical interpretation from those in (21.6). However, since the photon dynamics which interests us depends only on where the mirror M is located, this distinction makes no difference for the present analysis. Combining (21.14) with (21.2) gives an overall unitary time development of the photon and the mirror (and associated apparatus) from t_1 to t_2 in the form:

$$\begin{aligned} |1c\rangle|M_0\rangle &\mapsto \big(|2c\rangle|M_{out}\rangle + |2g\rangle|M_{in}\rangle\big)/\sqrt{2}, \\ |1d\rangle|M_0\rangle &\mapsto |2d\rangle\big(|M_{out}\rangle + |M_{in}\rangle\big)/\sqrt{2}. \end{aligned} \tag{21.15}$$

What is important is the location of the mirror at the time $t_{1.5}$ when the photon interacts with it—assuming both the mirror and the photon are in the c arm of the interferometer—and not its location in the initial state $|M_0\rangle$; the latter could be either the M_{in} or M_{out} position, or someplace else.

Let the initial state of the entire system at t_0 be

$$|\Omega_0\rangle = |0a\rangle|M_0\rangle|E^\circ\rangle|F^\circ\rangle|G^\circ\rangle, \tag{21.16}$$

and let $|\Omega_j\rangle$ be the state which results at time t_j from unitary time development. We assume that $|M_0\rangle$, $|M_{out}\rangle$ and $|M_{in}\rangle$ do not change outside the time interval where (21.14) and (21.15) apply. At t_3 one has

$$\begin{aligned} |\Omega_3\rangle = &\Big[\big(-|3e\rangle + |3f\rangle + \sqrt{2}|3g\rangle\big)|M_{in}\rangle + 2|3f\rangle|M_{out}\rangle\Big] \\ &\otimes |E^\circ\rangle|F^\circ\rangle|G^\circ\rangle/2\sqrt{2}. \end{aligned} \tag{21.17}$$

We leave to the reader the task of working out $|\Omega_j\rangle$ at other times, a useful exercise if one wants to check the properties of the various consistent families described below.

Corresponding to (21.10) there is a family, now based on the single initial state Ω_0, with support

$$\Omega_0 \odot [1\bar{a}] \odot \begin{cases} M_{out} \odot [3f] \odot F^*, \\ M_{in} \odot [3s] \odot \{E^*, F^*, G^*\}, \end{cases} \tag{21.18}$$

where $|3s\rangle$ is defined in (21.11). This confirms the fact that whether or not a photon arrives at E^* depends on the position of the mirror M at the time when the photon reaches the corresponding position in the c arm of the interferometer, not on where M was at an earlier time, in accordance with what was stated in Sec. 21.1. Suppose that the photon has been detected in E^*. From (21.18) it is evident that the quantum coin toss resulted in M_{in}. What would have happened if, instead, the result had been M_{out}? If we use $[1\bar{a}]$ at t_1 as a pivot, the answer is that the photon would have been detected by F. This is reasonable, but as noted in our discussion of (21.10), not at all paradoxical, since it is impossible to use this family to discuss whether or not the photon was in the d arm.

The counterpart of (21.12) is the family with support

$$\Omega_0 \odot \begin{cases} [1\bar{a}] \odot M_{out} \odot [3f] \odot F^*, \\ [1c] \odot \ M_{in} \ \odot [3g] \odot G^*, \\ [1d] \odot \ M_{in} \ \odot [3\bar{c}] \odot \{E^*, F^*\}. \end{cases} \tag{21.19}$$

Just as in (21.12), the photon states at t_1 are contextual; $[1c]$ and $[1d]$ depend on M_{in}, while $[1\bar{a}]$ depends on M_{out}. The only difference is that here the dependence is on the later, rather than earlier, position of the mirror M. Note once again that dependence, understood in the sense defined in Ch. 14, does not refer to a physical cause, and there is no reason to think that the future is influencing the past—see the discussion in Secs. 20.3 and 20.4. We can use (21.19) to conclude that the detection of the photon by E means that the photon was earlier in the d and not the c arm of the interferometer. However, due to the contextuality just mentioned, $[1d]$ at t_1 cannot serve as a pivot in a counterfactual argument which tells what would have happened had M_{out} occurred rather than M_{in}. The only pivot available in (21.19) is the initial state Ω_0. But the corresponding counterfactual assertion is too vague to serve as a satisfactory basis of a paradox, for precisely the same reasons given in Sec. 20.4 in connection with the analogous (20.23).

The counterpart of (21.13) is the family with support

$$\Omega_0 \odot \begin{cases} [1c] \odot \begin{cases} M_{out} \odot [3\bar{c}] \odot S^+, \\ M_{in} \odot [3g] \odot G^*, \end{cases} \\ [1d] \odot \begin{cases} M_{out} \odot [3\bar{d}] \odot S^-, \\ M_{in} \odot [3\bar{d}] \odot S^-. \end{cases} \end{cases} \tag{21.20}$$

Here $[1c]$ and $[1d]$ are no longer contextual. Also note that in this family there is not the slightest evidence of any nonlocal influence by the mirror on the photon: the later time development if the photon is in the d arm at t_1 is exactly the same for M_{in} and for M_{out}. However, (21.20) is clearly not a satisfactory formulation of the paradox, despite the fact that $[1d]$ at t_1 can serve as a pivot, Among other things, E^* does not appear at t_4. This can be remedied in part by replacing the fourth history in (21.20) with the two histories

$$\Omega_0 \odot [1d] \odot M_{in} \odot [3\bar{d}] \odot \{E^*, F^*\}. \tag{21.21}$$

The resulting family, now supported on five histories of non-zero weight, remains consistent. But E^* is a contextual event dependent on M_{in}, and if we use this family, E^* makes no sense in the case M_{out}. Thus, as noted above in connection with (21.13), we cannot when using this family employ E^* as evidence that the mirror was in rather than outside of the c arm. In addition to the difficulty just mentioned, (21.20) has MQS states at t_4. While one can modify the fourth history by replacing it with (21.21), the same remedy will not work in the other two cases, for it would violate the consistency conditions.

Let us summarize what we have learned from considering a situation in which a quantum coin toss at a time when the photon is already inside the interferometer determines whether or not the c arm will be blocked by the mirror M. For the photon to later be detected by E, it is necessary that M be in the c arm at the time when the photon arrives at this point, and in this respect E^* does, indeed, provide a (partial) measurement indicating M_{in} is the case rather than M_{out}. However, the attempt to infer from this that there is some sort of nonlocal influence between M and the photon fails, for reasons which are quite similar to those summarized at the end of Sec. 21.3: one needs to find a consistent family in which the photon is in the d arm both for M_{in} and for M_{out}. This is obviously not the case for (21.18) and (21.19), whereas (21.20)—with or without the fourth line replaced with (21.21)—is unsatisfactory because of the states which appear at t_4. Thus by the time one has constructed a family in which $[1d]$ at t_1 can serve as a pivot, the counterfactual analysis runs into difficulty because of what happens at later times. Just as in Sec. 21.3, one can construct various pieces of a paradox by using different consistent families. But the fact that these families are mutually incompatible prevents putting the pieces together to complete the paradox.

21.5 Interaction-Free Measurement?

It is sometimes claimed that the determination of whether M is blocking the c arm by means of a photon detected in E is an "interaction-free measurement": The photon did not actually interact with the mirror, but nonetheless provided information about its location. The term "interact with" is not easy to define in quantum theory, and we will want to discuss two somewhat different reasons why one might suppose that such an indirect measurement involves no interaction. The first is based on the idea that detection by E implies that the photon was earlier in the d arm of the interferometer, and thus far from the mirror and unable to interact with it—unless, of course, one believes in the existence of some mysterious long-range interaction. The second comes from noting that when it is in the c arm, Fig. 21.1(a), the mirror M is oriented in such a way that any photon hitting it will later be detected by G. Obviously a photon detected by E was not detected by G, and thus, according to this argument, could not have interacted with M. The consistent families introduced earlier are useful for discussing both of these ideas.

Let us begin with (21.10), or its counterpart (21.18) if a quantum coin is used. In these families the time development of the photon state is given by unitary transformations until it has been detected. As one would expect, the photon state is different, at times t_2 and later, depending upon whether M is in or out of the c arm. Hence if unitary time development reflects the presence or absence of some interaction, these families clearly do not support the idea that during the process which eventually results in E^* the photon does not interact with M. Indeed, one comes to precisely the opposite conclusion.

Suppose one considers families of histories in which the photon state evolves in a stochastic, rather than a unitary fashion preceding the final detection. Are the associated probabilities affected by the presence or absence of M in the c arm? In particular, can one find cases in which certain probabilities are the same for both M_{in} and M_{out}? Neither (21.12) nor its quantum coin counterpart (21.19) provide examples of such invariant probabilities, but (21.20) does supply an example: if the photon is in the d arm at t_1, then it will certainly be in the superposition state $[3\bar{d}]$ just after leaving the interferometer, and at a slightly later time the detector system will be in the MQS state S^-. (One would have the same thing in (21.13) if the last two histories were collapsed into a single history representing unitary time development after t_1, ending in S^- at t_4.) So in this case we have grounds to say that there was no interaction between the photon and the mirror if the photon was in the d arm at t_1. However, (21.20), for reasons noted in Sec. 21.4, cannot be used if one wants to speak of photon detection by E as representing a measurement of M_{in} as against M_{out}. Thus we have found a case which is "interaction free", but it cannot be called a "measurement".

Finally, let us consider the argument for non-interaction based upon the idea that had it interacted with the mirror, the photon would surely have been scattered into channel g to be detected by G. This argument would be plausible if we could be sure that the photon was in or not in the c arm of the interferometer at the time when it (might have) interacted with M. However, if the photon was in a superposition state at the relevant time, as is the case in the families (21.10) and (21.18), the argument is no longer compelling. Indeed, one could say that the M_{in} histories in these families provide a counterexample showing that when a quantum particle is in a delocalized state, a local interaction can produce effects which are contrary to the sort of intuition one builds up by using examples in classical physics, where particles always have well-defined positions.

In conclusion, there seems to be no point of view from which one can justify the term "interaction-free measurement". The one that comes closest might be that based on the family (21.20), in which the photon can be said to be definitely in the c or d arm of the interferometer, and when in the d arm it is not influenced by whether M is or is not in the c arm. But while this family can be used to argue for the absence of any mysterious long-range influences of the mirror on the photon, it is incompatible with using detection of the photon by E as a measurement of M_{in} in contrast to M_{out}.

It is worthwhile comparing the indirect measurement situation considered in this chapter with a different type of "interaction free" measurement discussed in Sec. 12.2 and in Secs. 18.1 and 18.2: A particle (photon or neutron) passes through a beam splitter, and because it is *not* detected by a detector in one of the two output channels, one can infer that it left the beam splitter through the other channel. In this situation there actually is a consistent family, see (12.31) or the analogous (18.7), containing the measurement outcomes, and in which the particle is far away from the detector in the case in which it is not detected. Thus one might have some justification for referring to this as "interaction-free". However, since such a situation can be understood quite simply in classical terms, and because "interaction-free" has generally been associated with confused ideas of wave function collapse, see Sec. 18.2, even in this case the term is probably not very helpful.

21.6 Conclusion

The paradox stated in Sec. 21.1 was analyzed by assuming, in Sec. 21.3, that the mirror positions M_{in} and M_{out} are specified at the initial time t_0, before the photon enters the interferometer, and then in Sec. 21.4 by assuming these positions are determined by a quantum coin toss which takes place when the photon is already inside the interferometer. Both analyses use several consistent families, and come to basically similar conclusions. In particular, while various parts of the argument leading to the paradoxical result—e.g., the conclusion that detection by E means the photon was earlier in the d arm of the interferometer—can be supported by choosing an appropriate framework, it is not possible to put all the pieces together within a single consistent family. Thus the reasoning which leads to the paradox, when restated in a way which makes it precise, violates the single framework rule.

This indicates a fourth lesson on how to analyze quantum paradoxes, which can be added to the three in Sec. 20.5. Very often quantum paradoxes rely on reasoning which violates the single framework rule. Sometimes such a violation is already evident in the way in which a paradox is stated, but in other instances it is more subtle, and analyzing several different frameworks may be necessary in order to discover where the difficulty lies.

The idea of a mysterious nonlocal influence of the position of mirror M (M_{in} vs. M_{out}) on the photon when the latter is far away from M in the d arm of the interferometer is not supported by a consistent quantum analysis. In the family (21.20) the absence of any influence is quite explicit. In the family (21.19) the fact that the photon states inside the interferometer are contextual events indicates that the difference between the photon states arises not from some physical influence of the mirror position, but rather from the physicist's choice of one form of description rather than another. (We found a very similar sort of "influence" of B_{in} and B_{out} in the delayed choice paradox of Ch. 20, and the remarks made there in Secs. 20.3 and 20.4 also apply to the indirect measurement paradox.) It is, of course, important to distinguish differences arising simply because one employs a different way of describing a situation from those which come about due to genuine physical influences.