Chapter 19

Coins and Counterfactuals

19.1 Quantum Paradoxes

The next few chapters are devoted to resolving a number of quantum paradoxes in the sense of giving a reasonable explanation of a seemingly paradoxical result in terms of the principles of quantum theory discussed earlier in this book. None of these paradoxes indicates a defect in quantum theory. Instead, when they have been properly understood, they show us that the quantum world is rather different from the world of our everyday experience and of classical physics, in a way somewhat analogous to that in which relativity theory has shows us that the laws appropriate for describing the behavior of objects moving at high speed differ in significant ways from those of pre-relativistic physics.

An inadequate theory of quantum measurements is at the root of several quantum paradoxes. In particular, the notion that wave function collapse is a physical effect produced by a measurement, rather than a method of calculation, see Sec. 18.2, has given rise to a certain amount of confusion. Smuggling rules for classical reasoning into the quantum domain where they do not belong and where they give rise to logical inconsistencies is another common source of confusion. In particular, many paradoxes involve mixing the results from incompatible quantum frameworks.

Certain quantum paradoxes have given rise to the idea that the quantum world is permeated by mysterious influences that propagate faster than the speed of light, in conflict with the theory of relativity. They are mysterious in that that they cannot be used to transmit signals, which means that they are, at least in any direct sense, experimentally unobservable. While relativistic quantum theory is outside the scope of this book, an analysis of non-relativistic versions of some of the paradoxes which are supposed to show the presence of superluminal influences indicates that the real source of such ghostly effects is the need to correct logical errors arising from the assumption that the quantum world is behaving in some respects in a classical way. When the situation is studied using consistent quantum principles, the ghosts disappear, and with them the corresponding difficulty in reconciling quantum mechanics with relativity theory. The reason why ghostly influences cannot be used to transmit signals faster than the speed of light is then obvious: there are no such influences.

Some quantum paradoxes are stated in a way that involves a free choice on the part of a human observer: e.g., whether to measure the x or the z component of spin angular momentum of some particle. Since the principles of quantum theory as treated in this book apply to a *closed system*, with all parts of it subject to quantum laws, a complete discussion of such paradoxes would require including the human observer as part of the quantum system, and using a quantum model of conscious human choice. This would be rather difficult to do given the current primitive state of scientific understanding of human consciousness. Fortunately, for most quantum paradoxes it seems possible to evade the issue of human consciousness by letting the outcome of a quantum coin toss "decide" what will be measured. As discussed in Sec. 19.2 below, the quantum coin is a purely physical device connected to a suitable servomechanism. By this means the stochastic nature of quantum mechanics can be used as a tool to model something which is indeterminate, which cannot be known in advance.

Certain quantum paradoxes are stated in terms of *counterfactuals*: what *would* have happened *if* some state of affairs had been different from what it actually was. Other paradoxes have both a counterfactual as well as in an "ordinary" form. In order to discuss counterfactual quantum paradoxes, one needs a quantum version of counterfactual reasoning. Unfortunately, philosophers and logicians have yet to reach agreement on what constitutes valid counterfactual reasoning in the classical domain. Our strategy will be to avoid the difficult problems which perplex the philosophers, such as "Would a kangaroo topple if it had no tail?", and focus on a rather select group of counterfactual questions which arise in a probabilistic context. These are of the general form: "What would have happened if the coin flip had resulted in heads rather than tails?" They are considered first from a classical (or everyday world) perspective in Sec. 19.3, and then translated into quantum terms in Sec. 19.4.

19.2 Quantum Coins

In a world governed by classical determinism there are no truly random events. But quantum mechanics allows for events which are irreducibly probabilistic. For example, a photon is sent into a beamsplitter and detected by one of two detectors situated on the two output channels. Quantum theory allows us to assign a probability that one detector or the other will detect the photon, but provides no deterministic prediction of which detector will do so in any particular realization of the experiment. This system generates a random output in the same way as tossing a coin, which is why it is reasonable to call it a quantum coin. One can arrange things so that the probabilities for the two outcomes are not the same, or so that there are three or even more random outcomes, with equal or unequal probabilities. We shall use the term "quantum coin" to refer to any such device, and "quantum coin toss" to refer to the corresponding stochastic process. There is no reason in principle why various experiments involving statistical sampling (such as drug trials) should not be carried out using the "genuine randomness" of quantum coins.

To illustrate the sort of thing we have in mind, consider the gedanken experiment in Fig. 19.1, in which a particle, initially in a wave packet $|0a\rangle$, is approaching a point P where a beam splitter B may may or may not be located depending upon the outcome of tossing a quantum coin Q shortly before the particle arrives at P. If the outcome of the toss is Q', the beam splitter is left in place at B', whereas if it is Q'', a servomechanism rapidly moves the beam splitter to B'' out of the path of the particle, which continues in a straight line.

Let us describe this in quantum terms in the following way. Suppose that $|Q\rangle$ is the initial state

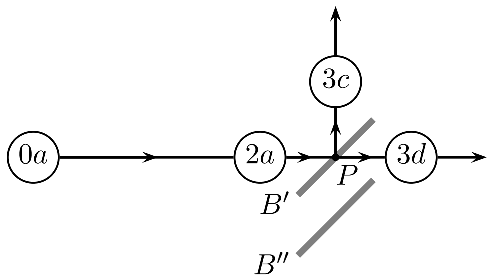

Figure 19.1: Particle paths approaching and leaving a beam splitter which is either left in place, B', or moved out of the way, B'', before the arrival of the particle.

of the quantum coin and the attached servomechanism at time t_0, and that between t_0 and t_1 there is a unitary time evolution

$$|Q\rangle \mapsto (|Q'\rangle + |Q''\rangle)/\sqrt{2}. \tag{19.1}$$

Next, let $|B'\rangle$ and $|B''\rangle$ be states corresponding to the beam splitter being either left in place or moved out of the path of the particle, and assume a unitary time evolution

$$|Q'\rangle|B'\rangle \mapsto |Q'\rangle|B'\rangle, \;\; |Q''\rangle|B'\rangle \mapsto |Q''\rangle|B''\rangle \tag{19.2}$$

between t_1 and t_2. Finally, the motion of the particle from t_2 to t_3 is governed by

$$|2a\rangle|B'\rangle \mapsto (|3c\rangle + |3d\rangle)|B'\rangle/\sqrt{2}, \;\; |2a\rangle|B''\rangle \mapsto |3d\rangle|B''\rangle, \tag{19.3}$$

where $|2a\rangle$ is a wave packet on path a for the particle at time t_2, and a similar notation is used for wave packets on paths c and d in Fig. 19.1. The overall unitary time evolution of the system consisting of the particle, the quantum coin, and the apparatus during the time interval from t_0 until t_3 takes the form

$$\begin{aligned} |\Psi_0\rangle = |0a\rangle \otimes |Q\rangle|B'\rangle &\mapsto |1a\rangle \otimes \big(|Q'\rangle + |Q''\rangle\big)|B'\rangle/\sqrt{2} \\ &\mapsto |2a\rangle \otimes \big(|Q'\rangle|B'\rangle + |Q''\rangle|B''\rangle\big)/\sqrt{2} \\ &\mapsto \big(|3c\rangle + |3d\rangle\big) \otimes |Q'\rangle|B'\rangle/2 + |3d\rangle \otimes |Q''\rangle|B''\rangle/\sqrt{2}, \end{aligned} \tag{19.4}$$

where $\otimes$ helps to set the particle off from the rest of the quantum state.

There are reasons, discussed in Sec. 17.4, why macroscopic objects are best described not with individual kets but with macro projectors, or statistical distributions or density matrices. The use of kets is not misleading, however, and it makes the reasoning somewhat simpler. With a little effort—again, see Sec. 17.4—one can reconstruct arguments of the sort we shall be considering so that macroscopic properties are represented by macro projectors. While we will continue to use the simpler arguments, projectors representing macroscopic properties will be denoted by symbols without square brackets, as in (19.5) below, so that the formulas remain unchanged in a more sophisticated analysis.

Consider the consistent family for the times $t_0 < t_1 < t_2 < t_3$ with support consisting of the two histories

$$\Psi_0 \odot \begin{cases} Q' \odot B' \odot [3\bar{a}], \\ Q'' \odot B'' \odot [3d], \end{cases} \tag{19.5}$$

where

$$|3\bar{a}\rangle := \big(|3c\rangle + |3d\rangle\big)/\sqrt{2}. \tag{19.6}$$

It allows one to say that if the quantum coin outcome is Q', then the particle is later in the coherent superposition state $|3\bar{a}\rangle$, a state which could be detected by bringing the beams back together again and passing them through a second beam splitter, as in Fig. 18.3(c). On the other hand, if the outcome is Q'', then the particle will later be in channel d in a wavepacket $|3d\rangle$. As $[3\bar{a}]$ and $[3d]$ do not commute with each other, it is clear that these final states in (19.5) are dependent, in the sense discussed in Ch. 14, either upon the earlier beam splitter locations $|B'\rangle$ and $|B''\rangle$, or the still earlier outcomes $|Q'\rangle$ and $|Q''\rangle$ of the quantum coin toss.

The expressions in (19.4) are a bit cumbersome, and the same effect can be achieved with a somewhat simpler notation in which (19.1) and (19.2) are replaced by the single expression

$$|B_0\rangle \mapsto \big(|B'\rangle + |B''\rangle\big)/\sqrt{2}, \tag{19.7}$$

where $|B_0\rangle$ is the initial state of the entire apparatus, including the quantum coin and the beam splitter, whereas $|B'\rangle$ and $|B''\rangle$ are apparatus states in which the beam splitter is at the locations B' and B'' indicated in Fig. 19.1. The time development of the particle in interaction with the beam splitter is given, as before, by (19.3).

19.3 Stochastic Counterfactuals

A workman falls from a scaffolding, but is caught by a safety net, so he is not injured. What *would* have happened *if* the safety net had not been present? This is an example of a *counterfactual* question, where one has to imagine something different from what actually exists, and then draw some conclusion. Answering it involves counterfactual reasoning, which is employed all the time in the everyday world, though it is still not entirely understood by philosophers and logicians. In essence it involves comparing two or more possible states of affairs, often referred to as "worlds", which are similar in certain respects and differ in others. In the example just considered, a world in which the safety net is present is compared to a world in which it is absent, while both worlds have in common the feature that the workman falls from the scaffolding.

We begin our study of counterfactual reasoning by looking at a scheme which is able to address a limited class of counterfactual questions in a *classical* but *stochastic* world, that is, one in which there is a random element added to classical dynamics. The world of everyday experience is such a world, since classical physics gives deterministic answers to some questions, but there are others, e.g., "What will the weather be two weeks from now?", for which only probabilistic answers are available.

Shall we play badminton or tennis this afternoon? Let us toss a coin: H (heads) for badminton, T (tails) for tennis. The coin turns up T, so we play tennis. What *would* have happened *if* the result of the coin toss had been H? It is useful to introduce a diagrammatic way of representing the question and deriving an answer, Fig. 19.2. The node at the left at time t_1 represents the situation before the coin toss, and the two nodes at t_2 are the mutually exclusive possibilities resulting from that toss. The lower branch represents what actually occurred: the toss resulted in T and a game of tennis. To answer the question of what would have happened if the coin had turned up the other way, we start from the node representing what actually happened, go backwards in time to the node preceding the coin toss, which we shall call the *pivot*, and then forwards along the alternative branch to arrive at the badminton game. This type of counterfactual reasoning can be thought of as comparing histories in two "worlds" which are identical at all times up to and including the pivot point t_1 at which the coin is tossed. After that, one of these worlds contains the outcome H and the consequences which flow from this, including a game of badminton, while the other world contains the outcome T and its consequences.

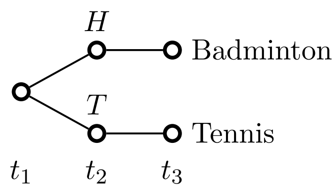

Figure 19.2: Diagram for counterfactual analysis of a coin toss.

It is instructive to embed the preceding example in a slightly more complicated situation. Let us suppose that the choice between tennis or badminton was preceded by another: should we go visit the museum, or get some exercise? Once again, imagine the decision being made by tossing a coin at time t_0, with H leading to exercise and T to a museum visit. At the museum a choice between visiting one of two exhibits can also be carried out by tossing a coin. The set of possibilities is shown in Fig. 19.3. Suppose that the actual sequence of the two coins was H_1T_2, leading to tennis. If the first coin toss had resulted in T_1 rather than H_1, what would have happened? Start from the tennis node in Fig. 19.3, go back to the pivot node P_0 at t_0 preceding the first coin toss, and then forwards on the alternative, T_1 branch. This time there is not a unique possibility, for the second coin toss could have been either H_2 or T_2. Thus the appropriate answer would be: Had the first coin toss resulted in T_1, we would have gone to one or the other of the two exhibits at the museum, each possibility having probability one half. That counterfactual questions have probabilistic answers is just what one would expect if the dynamics describing the situation is stochastic, rather than deterministic. The answer is deterministic only in the limiting cases of probabilities equal to 1 or 0.

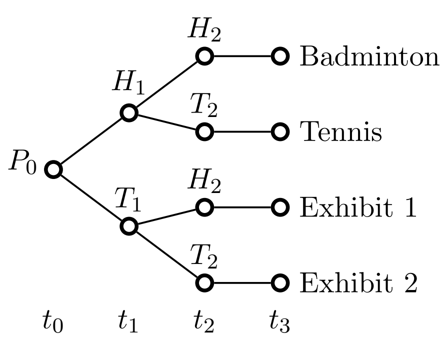

Figure 19.3: Diagram for analyzing two successive coin tosses.

However, a somewhat surprising feature of stochastic counterfactual reasoning comes to light if we ask the question, again assuming the afternoon was devoted to tennis, "What would have happened if the first coin had turned up H_1 (as it actually did)?", and attempt to answer it using the diagram in Fig. 19.3. Let us call this a *null counterfactual* question, since it asks not what would have happened if the world had been different in some way, but what would have happened if the world had been the same in this particular respect. The answer obtained by tracing from "tennis" backwards to P_0 in Fig. 19.3 and then forwards again along the upper, or H_1 branch, is not tennis, but it is badminton or tennis, each with probability one half. We do not, in other words, reach the conclusion that what actually happened would have happened had the world been the same in respect to the outcome of the first coin toss. Is it reasonable to have a stochastic answer, with probability less than one, for a null counterfactual question? Yes, because to have a deterministic answer would be to specify implicitly that the second coin toss turned out the way it actually did. But in a world which is not deterministic there is no reason why random events should not have turned out differently.

Counterfactual questions are sometimes ambiguous because there is more than one possibility for a pivot. For example, "What would we have done if we had not played tennis this afternoon?" will be answered in a different way depending upon whether H_1 or P_0 in Fig. 19.3 is used as the pivot. In order to make a counterfactual question precise, one must specify both a framework of possibilities, as in Fig. 19.3, and also a pivot, the point at which the actual and counterfactual worlds, identical at earlier times, "split apart".

This method of reasoning is useful for answering some types of counterfactual questions but not others. Even to use it for the case of a workman whose fall is broken by a safety net requires an exercise in imagination. Let us suppose that just after the workman started to fall (the pivot), the safety net was swiftly removed, or left in place, depending upon some rapid electronic coin toss, so that the situation could be represented in a diagram similar to Fig. 19.2. Is this an adequate, or at least a useful way of thinking about this counterfactual question? At least it represents a way to get started, and we shall employ the same idea for quantum counterfactuals.

19.4 Quantum Counterfactuals

Counterfactuals have played an important role in discussions of quantum measurements. Thus a perennial question in the foundations of quantum theory is whether measurements reveal pre-existing properties of a measured system, or whether they somehow "create" such properties. Suppose, to take an example, that a Stern-Gerlach measurement reveals the value $S_x = 1/2$ for a spin half particle. Would the particle have had the same value of S_x even if the measurement had not been made? An interpretation of quantum theory which gives a "yes" answer to this counterfactual question can be said to be *realistic* in that it affirms the existence of certain properties or events in the world independent of whether measurements are made. (For some comments on realism in quantum theory, see Ch. 27.) Another similar counterfactual question is the following: Given that the S_x measurement outcome indicates, using an appropriate framework (see Ch. 17), that the value of S_x was $+1/2$ before the measurement, would this still have been the case if S_z had been measured instead of S_x?

The system of quantum counterfactual reasoning presented here is designed to answer these and similar questions. It is quite similar to that introduced in the previous section for addressing classical counterfactual questions. It makes use of quantum coins of the sort discussed in Sec. 19.2, and diagrams like those in Figs. 19.2 and 19.3. The nodes in these diagrams represent events in a consistent family of quantum histories, and nodes connected by lines indicate the histories with finite weight that form the support of the family. We require that the family be consistent, and that *all* the histories in the diagram belong to the *same* consistent family. This is a *single framework* rule for quantum counterfactual reasoning comparable to the one discussed in Sec. 16.1 for ordinary quantum reasoning.

Let us see how this works in the case in which S_x is the component of spin actually measured for a spin-half particle, and we are interested in what would have been the case if S_z had been measured instead. Imagine a Stern-Gerlach apparatus of the sort discussed in Sec. 17.2 or Sec. 18.3, arranged so that it can be rotated about an axis (in the manner indicated in Sec. 18.3) to measure either S_x or S_z. When ready to measure S_x its initial state is $|X^\circ\rangle$, and its interaction with the particle results in the unitary time development

$$|x^+\rangle \otimes |X^\circ\rangle \mapsto |X^+\rangle, \quad |x^-\rangle \otimes |X^\circ\rangle \mapsto |X^-\rangle. \tag{19.8}$$

Similarly, when oriented to measure S_z the initial state is $|Z^\circ\rangle$, and the corresponding time development is

$$|z^+\rangle \otimes |Z^\circ\rangle \mapsto |Z^+\rangle, \quad |z^-\rangle \otimes |Z^\circ\rangle \mapsto |Z^-\rangle. \tag{19.9}$$

The symbols X°, etc., without square brackets will be used to denote the corresponding projectors. Because they refer to macroscopically distinct states, all the Z projectors are orthogonal to all the X projectors: $X^+Z^+ = 0$, etc. Without loss of generality we can consider the quantum coin and the associated servomechanism to be part of the Stern-Gerlach apparatus, which is initially in the state $|A\rangle$, with the coin toss corresponding to a unitary time development

$$|A\rangle \mapsto \big(|X^\circ\rangle + |Z^\circ\rangle\big)/\sqrt{2}. \tag{19.10}$$

Assume that the spin-half particle is prepared in an initial state $|w^+\rangle$, where the exact choice of w is not important for the following discussion, provided it is not $+x$, $-x$, $+z$, or $-z$. Suppose that X^+ is observed: the quantum coin resulted in the apparatus state X° appropriate for a measurement of S_x, and the outcome of the measurement corresponds to $S_x = +1/2$. What would have happened if the quantum coin toss had, instead, resulted in the apparatus state Z° appropriate for a measurement of S_z? To address this question we must adopt some consistent family and identify the event which serves as the pivot. As in other examples of quantum reasoning, there is more than one possible family, and the answer given to a counterfactual question can depend upon which family one uses. Let us begin with a family whose support consists of the four histories

$$\Psi_0 \odot I \odot \begin{cases} X^\circ \odot \begin{cases} X^+, \\ X^-, \end{cases} \\ Z^\circ \odot \begin{cases} Z^+, \\ Z^-, \end{cases} \end{cases} \tag{19.11}$$

at the times $t_0 < t_1 < t_2 < t_3$, where $|\Psi_0\rangle = |w^+\rangle \otimes |A\rangle$ is the initial state. It is represented in Fig. 19.4 in a diagram resembling those in Figs. 19.2 and 19.3. The quantum coin toss (19.10) takes place between t_1 and t_2. The particle reaches the Stern-Gerlach apparatus and the measurement occurs between t_2 and t_3, and at t_3 the outcome of the measurement is indicated by one of the four pointer states (end of Sec. 9.5) $X^{\pm}$, $Z^{\pm}$. Notice that only the first branching in Fig. 19.4, between t_1 and t_2, corresponds to the alternative outcomes of the quantum coin toss, while the later branching is due to other stochastic quantum processes.

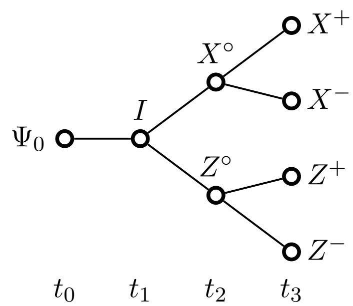

Figure 19.4: Diagram for counterfactual analysis of the family (19.11).

Suppose S_x was measured with the result X^+. To answer the question of what would have occurred if S_z had been measured instead, start with the X^+ vertex in Fig. 19.4, trace the history back to I at t_1 (or Ψ_0 at t_0) as a pivot, and then go forwards on the lower branch of the diagram through the Z° node. The answer is that one of the two outcomes Z^+ or Z^- would have occurred, each possibility having a positive probability which depends on w, which seems reasonable. Rather than using the nodes in Fig. 19.4, one can equally well use the support of the consistent family written in the form (19.11), as there is an obvious correspondence between the nodes in the former and positions of the projectors in the latter. From now on we will base counterfactual reasoning on expressions of the form (19.11), interpreted as diagrams with nodes and lines in the fashion indicated in Fig. 19.4.

Now ask a different question. Assuming, once again, that X^+ was the actual outcome, what would have happened if the quantum coin had resulted (as it actually did) in X° and thus a measurement of S_x? To answer this null counterfactual question, we once again trace the actual history in (19.11) or Fig. 19.4 backwards from X^+ at t_3 to the I or the Ψ_0 node, and then forwards again along the upper branch through the X° node at t_2, since we are imagining a world in which the quantum coin toss had the same result as in the actual world. The answer to the question is that either X^+ or X^- would have occurred, each possibility having some positive probability. Since quantum dynamics is intrinsically stochastic in ways which are not limited to a quantum coin toss, there is no reason to suppose that what actually did occur, X^+, would necessarily have occurred, given only that we suppose the same outcome, X° rather than Z°, for the coin toss.

Nevertheless, it is possible to obtain a more definitive answer to this null counterfactual question by using a different consistent family with support

$$\Psi_0 \odot \begin{cases} [x^+] \odot \begin{cases} X^\circ \odot X^+, \\ Z^\circ \odot U^+, \end{cases} \\ [x^-] \odot \begin{cases} X^\circ \odot X^-, \\ Z^\circ \odot U^-, \end{cases} \end{cases} \tag{19.12}$$

where the nodes $[x^{\pm}]$ at t_1, a time which precedes the quantum coin toss, correspond to the spin states $S_x = \pm 1/2$, and U^+ and U^- are defined in the next paragraph. The history which results in X^+ can be traced back to the pivot $[x^+]$, and then forwards again along the same (upper) branch, since we are assuming that the quantum coin toss in the alternative (counterfactual) world did result in the X° apparatus state. The result is X^+ with probability one. That this is reasonable can be seen in the following way. The actual measurement outcome X^+ shows that the particle had $S_x = +1/2$ at time t_1 before the measurement took place, since quantum measurements reveal pre-existing values if one employs a suitable framework. And by choosing $[x^+]$ at t_1 as the pivot, one is assuming that S_x had the same value at this time in both the actual and the counterfactual world. Therefore a later measurement of S_x in the counterfactual world would necessarily result in X^+.

However, we find something odd if we use (19.12) to answer our earlier counterfactual question of what would have happened if S_z had been measured rather than S_x. Tracing the actual history backwards from X^+ to $[x^+]$ and then forwards along the lower branch in the upper part of (19.12), through Z°, we reach U^+ at t_3 rather than the pair Z^+, Z^-, as in (19.11) or Fig. 19.4. Here U^+ is a projector on the state $|U^+\rangle$ obtained by *unitary* time evolution of $|x^+\rangle|Z^\circ\rangle$ using (19.9):

$$|x^+\rangle|Z^\circ\rangle = \big(|z^+\rangle + |z^-\rangle\big)|Z^\circ\rangle/\sqrt{2} \mapsto |U^+\rangle = \big(|Z^+\rangle + |Z^-\rangle\big)/\sqrt{2}. \tag{19.13}$$

Similarly, U^- in (19.12) projects on the state obtained by unitary time evolution of $|x^-\rangle|Z^\circ\rangle$. Both U^+ and U^- are macroscopic quantum superposition (MQS) states. The appearance of these MQS states in (19.12) reflects the need to construct a family satisfying the consistency conditions, which would be violated were we to use the pointer states Z^+ and Z^- at t_3 following the Z° nodes at t_2. The fact that consistency conditions sometimes require MQS states rather than pointer states is significant for analyzing certain quantum paradoxes, as we shall see in later chapters.

The contrasting results obtained using the families in (19.11) and (19.12) illustrate an important feature of quantum counterfactual reasoning of the type we are discussing: the outcome depends upon the family of histories which is used, and also upon the pivot. In order to employ the pivot $[x^+]$ rather than I at t_1, it is necessary to use a family in which the former occurs, and it cannot simply be added to the family (19.11) by a process of refinement. To be sure, this dependence upon the framework and pivot is not limited to the quantum case; it also arises for classical stochastic counterfactual reasoning. However, in a classical situation the framework is a classical sample space with its associated event algebra, and framework dependence is rather trivial. One can always, if necessary, refine the sample space, which corresponds to adding more nodes to a diagram such as Fig. 19.3, and there is never a problem with incompatibility or MQS states.

Consider a somewhat different question. Suppose the actual measurement outcome corresponds to $S_x = +1/2$. Would S_x have had the same value if no measurement had been carried out? To address this question, we employ an obvious modification of the previous gedanken experiment, in which the quantum coin leads either to a measurement of S_x, as actually occurred, or to no measurement at all, by swinging the apparatus out of the way before the arrival of the particle. Let $|N\rangle$ denote the state of the apparatus when it has been swung out of the way. An appropriate consistent family is one with support

$$\Psi_0 \odot \begin{cases} [x^+] \odot \begin{cases} X^\circ \odot X^+, \\ N \odot [x^+], \end{cases} \\ [x^-] \odot \begin{cases} X^\circ \odot X^-, \\ N \odot [x^-]. \end{cases} \end{cases} \tag{19.14}$$

It resembles (19.12), but with Z° replaced by N, U^+ by $[x^+]$, and U^- by $[x^-]$, since if no measuring apparatus is present, the particle continues on its way in the same spin state.

We can use this family and the node $[x^+]$ at time t_1 to answer the question of what would have happened in a case in which the measurement result was $S_x = +1/2$ if, contrary to fact, no measurement had been made. Start with the X^+ node at t_3, trace it back to $[x^+]$ at t_1, and then forwards in time through the N node at t_2. The result is $[x^+]$, so the particle would have been in the state $S_x = +1/2$ at t_1 and at later times if no measurement had been made.