

Consistent/Decoherent Histories

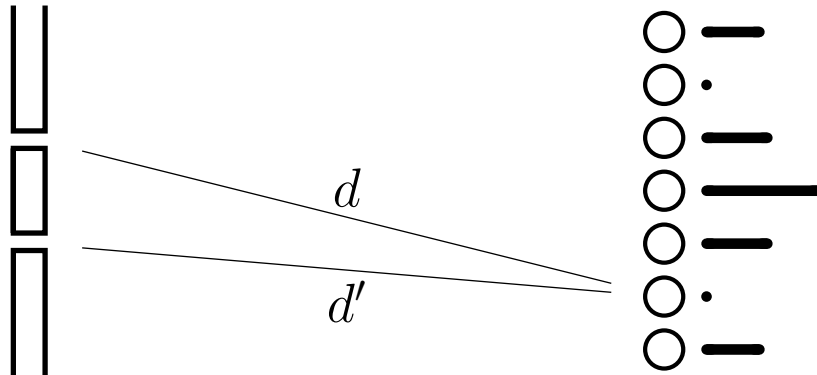
- Conceptual difficulties in QM come from introducing probabilities in the wrong way
- Histories approach: consistent introduction of probabilities eliminates difficulties and resolves (tames) paradoxes
- History of histories:
 - Griffiths 1984
 - Omnès 1987
 - Gell-Mann and Hartle 1990
 - Many subsequent papers, books
 - Griffiths, CONSISTENT QUANTUM THEORY (Cambridge 2002). First 12 chapters available at:
<http://quantum.phys.cmu.edu/CQT>

Histories and Paradoxes

- Paradoxes that are resolved/tamed using histories:
 - Einstein, Podolsky, Rosen
 - Double slit
 - Bell, Kochen, Specker
 - Greenberger, Horne, Zeilinger
 - Hardy
 - Aharonov and Vaidman multiple box
 - Wheeler delayed choice
 - Elitzur and Vaidman noninteracting measurement
 - ...

- Paradoxes that are not resolved using histories:
 - Any suggestions?

Double Slit I



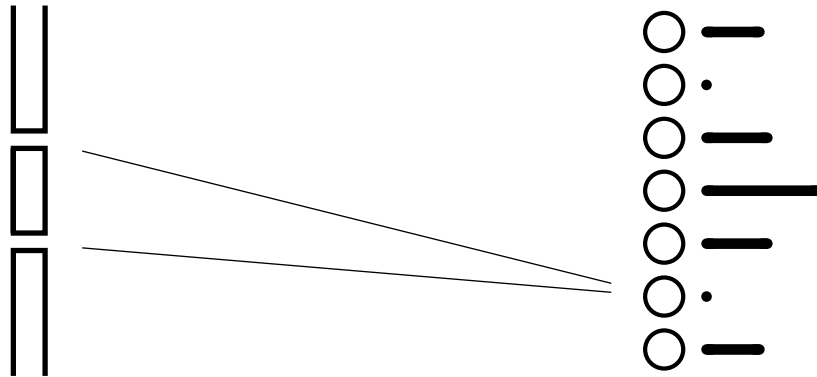
- Slit system, detectors in interference region
 - Horizontal bars: counting rates
 - Interference depends on *difference* $d - d'$, so
 - particles pass through slits *coherently*
 - Particles arrive *randomly* at detectors

- Consistent histories
 - Randomness an intrinsic part of nature
 - Anti-Einstein. There are no hidden variables

Histories and Measurements

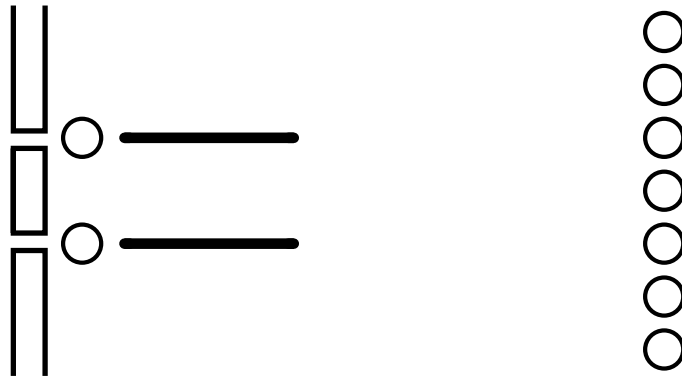
- Textbook QM:
 - Randomness arises through *measurements*
- Histories:
 - Randomness *intrinsic* in QM
 - Measurements are examples of physical processes
 - Same quantum principles govern *all* processes
 - There is *no* classical world, apparatus
 - *Sometimes* classical mechanics is a good approx
 - Quantum principles determine those circumstances

Double Slit II



- Experimentalist:
 - Detector triggers *because* particle arrives
 - Just *before* detection particle was near detector, on its way to detector
- Historian:
 - Good experimentalists know what they're doing
 - Triggered detector indicates arrival of particle
 - QM justifies this talk; indicates its limitations
 - Theorists should not bully competent people!

Double Slit III



- Detectors directly behind slits
 - Particles arrive at random
 - Total counting rate same as before
 - *One* detector, not both, detects each particle

- Explanations

- Experimentalist:

Particle came through slit preceding detector—

Collimators work this way

- Textbook:

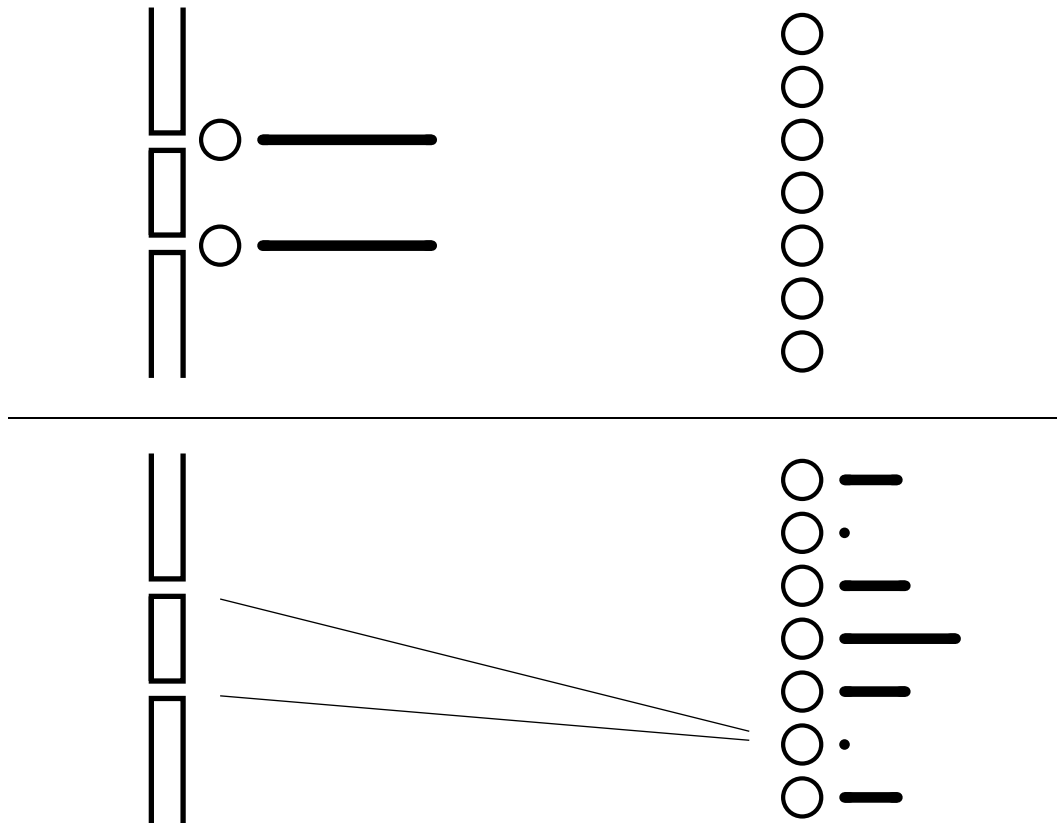
Cannot discuss what happened before measurement

“Great Smoky Dragon”

- Historian:

QM supports experimentalist account

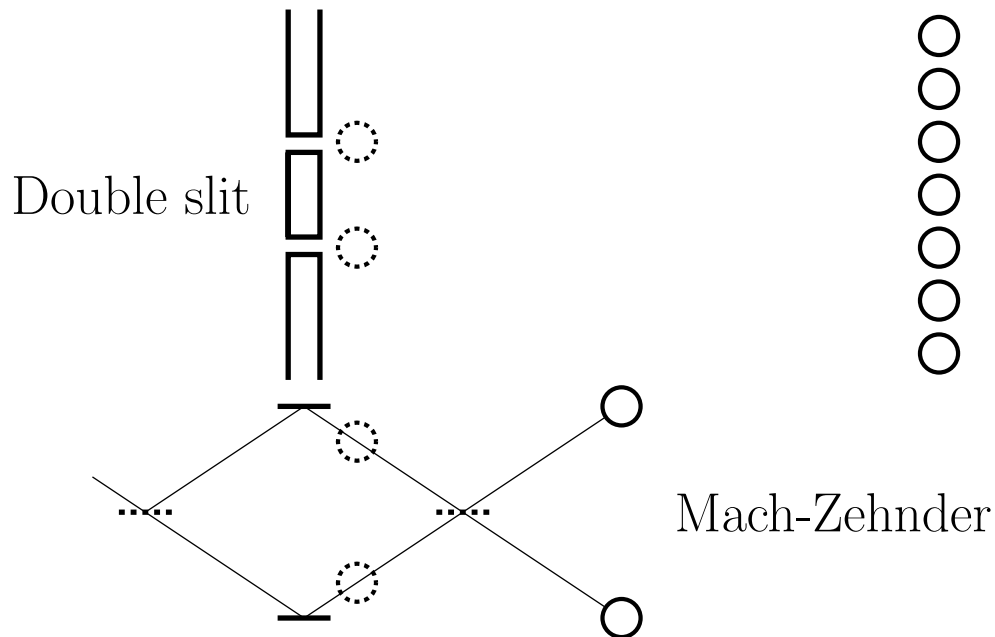
Double Slit IV



- Detectors behind slits removed at the last moment
 - Detectors remain:
 - particle came through definite slit
 - Detectors removed:
 - particle passed through slits coherently

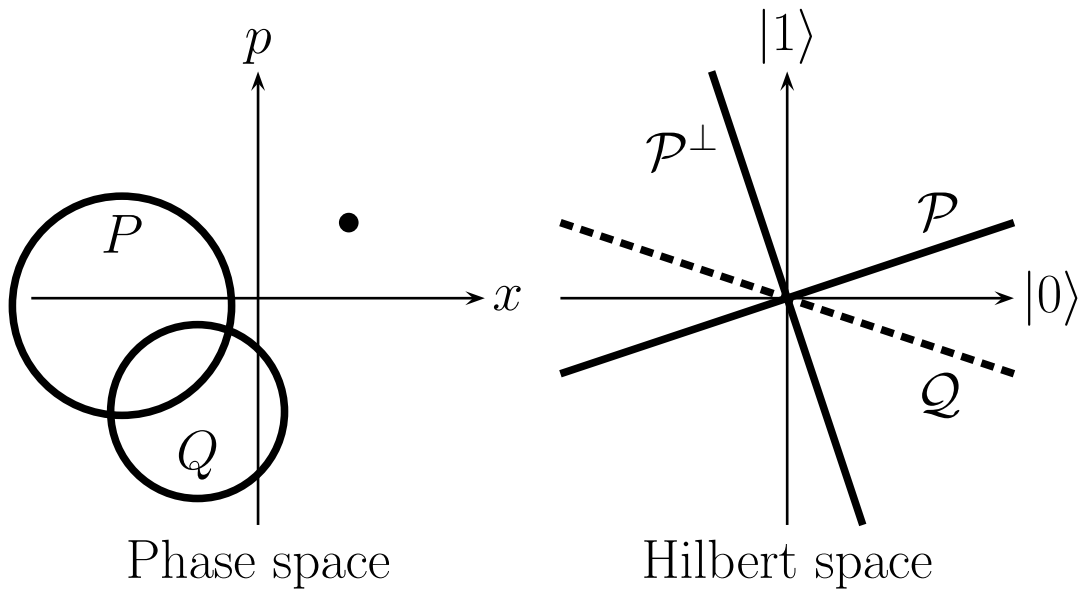
- Particle could enter slit system before decision to remove detectors was made! (Wheeler delayed choice)
 - Does the future influence the past?

Double Slit + Mach-Zehnder



- Correspondences:
 - Which slit? \leftrightarrow Which arm?
 - Detectors behind slits \leftrightarrow inside interferometer
 - In interference zone \leftrightarrow following 2d beam splitter
- For precise description, use Mach-Zehnder
 - Basic idea applies to double slit

Phase Space and Hilbert Space



	Classical	Quantum
Physical state	Point	Ray
Property P	Subset P	Subspace \mathcal{P}
NOT P	Compl. $\sim P$	Orthog. compl. \mathcal{P}^\perp
P AND Q	$P \cap Q$?

Spin Half Particle

- $S_z = +1/2$ is a physical state
 - Ray in Hilbert space. Point on Bloch sphere
- $S_z = -1/2$ is negation of $S_z = +1/2$
 - Orthogonal ray. Antipode on Bloch sphere
- For any spin-half particle,
 - Either $S_z = +1/2$ or $S_z = -1/2$, not both
 - Stern-Gerlach measurement shows which is the case

- Nothing special about z . The x axis is just as good.
- For any spin-half particle,
 - Either $S_x = +1/2$ or $S_x = -1/2$, not both
 - Stern-Gerlach measurement shows which is the case

- $S_z = +1/2$ AND $S_x = +1/2$ is *meaningless*:
 - Hilbert space QM assigns it no meaning
 - No corresponding ray in the Hilbert space
 - No experiment which can measure it
 - Because it is not there!

Logic of Quantum Properties

- False statement is one whose negation is true
 - “Pennsylvania is a Canadian province”
- Meaningless statement: not formed according to rules governing proper use of the language
 - Example: “ $P \wedge \vee Q$ ”
 - Negation of meaningless statement is meaningless
- Classical physical system:
 - Meaningful to combine two properties with AND
 - “The position is. . .” AND “The momentum is. . .”
- Quantum physical system:
 - Use AND only with *compatible* properties
 - Compatible: projectors commute: $PQ = QP$
 - $S_z = +1/2$, $S_x = +1/2$ are *incompatible*

Quantum Logic

- George Birkhoff and John von Neumann, Ann. Math. 37 (1936) 823, “The Logic of Quantum Mechanics”
 - $S_z = +1/2$ AND $S_x = +1/2$ is *meaningful, false*
 - Must modify rules of logic:

$$A \vee (B \wedge C) \neq (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C) \neq (A \wedge B) \vee (A \wedge C)$$
- Consistent histories recognizes logical problem
 - But solves it in a different way
 - $S_z = +1/2$ AND $S_x = +1/2$ is *meaningless*
- Rules of logic remain unchanged, but one must
 - Recognize and exclude meaningless statements
- Single framework rule:
 - Meaningful quantum descriptions use a *single collection* of mutually compatible properties
 - Incompatible descriptions cannot be combined!
- Spin half
 - Can discuss S_z , which is $+1/2$ or $-1/2$
 - Can discuss S_x , which is $+1/2$ or $-1/2$
 - Cannot *combine* these discussions
 - Doing so makes no sense in Hilbert space QM

Probabilities I

- Standard (textbook) probability theory: $(\mathcal{S}, \mathcal{E}, \text{Pr})$
- Sample space \mathcal{S} of *mutually-exclusive possibilities*
 - One and only one occurs in a given experiment
 - Examples:
 - $\{H, T\}$ for coin toss
 - $\{1, 2, 3, 4, 5, 6\}$ for roll of die
- Event algebra \mathcal{E} .
 - Assume \mathcal{S} discrete; $\mathcal{E} =$ all subsets of \mathcal{S}
- Probability distribution Pr
 - To each s_i in \mathcal{S} assign $p_i = \text{Pr}(s_i) \geq 0$; $\sum_i p_i = 1$.

- Quantum mechanics: three options for probabilities
 - (i) Use standard theory; (ii) Invent new one;
 - (iii) Become confused (very popular option)
- Consistent histories uses *standard* probability theory
 - There are two tasks:
 - Define quantum sample space \mathcal{S}
 - Introduce probabilities Pr

Probabilities II

- Example of spin half
- $S_z = +1/2, -1/2$ are mutually exclusive possibilities
 - If one is true, the other is false
 - One, only one occurs in Stern-Gerlach experiment
 - They constitute the S_z sample space
- Likewise, $S_x = +1/2, -1/2$ constitute S_x sample space
- S_z and S_x sample spaces are *incompatible*
 - Events cannot be combined
 - Probabilistic inference cannot be combined
- WARNING!
 - *Incompatible* is a *quantum* concept
 - *Mutually Exclusive* is classical or quantum
 - Do not confuse the two!

Probabilities III

- General structure of quantum sample spaces
- Decomposition of the identity in projectors $\{P^j\}$
 - (Superscript is label, not power)
 - $P^j = (P^j)^\dagger$, $P^j P^k = \delta_{jk} P^j$, $I = \sum_j P^j$
 - Each $P^j \leftrightarrow$ physical property (Hilbert subspace)
 - $j \neq k \Rightarrow P^j P^k = 0$: mutually exclusive properties
 - $\sum_j P^j = 1$: at least one property is true.
- Event algebra \mathcal{E} consists of all projectors of type

$$P = \sum_j \pi_j P^j, \quad \pi_j = 0 \text{ or } 1$$
- Example: Orthonormal basis $\{|\phi^j\rangle\}$; $P^j = |\phi^j\rangle\langle\phi^j|$.
- S_z sample space for spin half: $I = [z^+] + [z^-]$
 - Use $[\psi]$ as abbreviation for dyad $|\psi\rangle\langle\psi|$.
- Before discussing quantum probabilities, make sure sample space exists! Many quantum paradoxes and other confusion can be traced to nonexistent sample spaces!

Born Rule I

- Time development of *closed* or *isolated* physical system
 - Open system: make it part of larger closed system
 - Use Schrödinger Eqn to compute probabilities
 - Born rule is first (but not last!) step
- Unitary time development operator $T(t, t')$
 - Comes from solving Schrödinger's equation
 - Time-independent H : $T(t, t') = e^{-i(t-t')H/\hbar}$
- Assume $|\psi_0\rangle$ at t_0
 - Sample space \mathcal{S} : basis $\{|\phi_1^k\rangle\}$ at t_1

- Born probabilities:

$$\Pr(\phi_1^k) = \Pr(\phi_1^k | \psi_0) = |\langle \phi_1^k | T(t_1, t_0) | \psi_0 \rangle|^2$$

- $\Pr(\phi_1^k) = \text{prob of } [\phi_1^k], \text{ not measurement of } \phi_1^k.$
 - Good measurements reveal pre-existing properties.
 - Use *quantum* description *including* apparatus to discuss measurements

Born Rule II

- Born probabilities *depend on basis* $\{|\phi_1^k\rangle\}$
- Example. Spin half, $|\psi_0\rangle = |z^+\rangle$, $H = 0$, $T(t, t') = I$
- S_z basis $\{|z^+\rangle, |z^-\rangle\}$ at t_1 :
 - $\Pr(z_1^+) = 1$, $\Pr(z_1^-) = 0$
 - Subscript 1 indicates time t_1 .
- S_x basis $\{|x^+\rangle, |x^-\rangle\}$ at t_1 :
 - $\Pr(x_1^+) = 1/2 = \Pr(x_1^-)$
- Probabilities refer to properties of particle!
 - Bases incompatible; *cannot* assign probability to $S_z = +1/2$ AND $S_x = -1/2$ at time t_1
- “Gyroscope with axis in z direction” is misleading
 - $S_z = +1/2$ at t_0 , $S_x = -1/2$ at t_1 does *not* mean change in direction of axis!
 - Better picture: gyroscope axis in random direction
 - Given z component at t_0 , what is probability of x component at t_1 ?

Pre-Probability $|\psi_t\rangle$

- Born probability

$$\Pr(\phi_1^k) = \Pr(\phi_1^k | \psi_0) = |\langle \phi_1^k | T(t_1, t_0) | \psi_0 \rangle|^2$$

can be calculated in different ways.

- 1. Integrate Schrödinger Eqn from t_0 to t_1
 - $|\psi_1\rangle = T(t_1, t_0)|\psi_0\rangle$
 - $\Pr(\phi_1^k | \psi_0) = |\langle \phi_1^k | \psi_1 \rangle|^2$
- 2. Integrate Schrödinger Eqn from t_1 to t_0
 - $|\phi_0^k\rangle = T(t_0, t_1)|\phi_1^k\rangle$
 - $\Pr(\phi_1^k | \psi_0) = |\langle \phi_0^k | \psi_0 \rangle|^2$
- Approaches 1 and 2 equally good
 - Compare E&M: same result using different gauge
- Physical reality: $|\psi_0\rangle$ and the $\{|\phi_1^k\rangle\}$;
 - however, $|\psi_1\rangle$ and $\{|\phi_0^k\rangle\}$ are *pre-probabilities*:
tools for computing probabilities, *not* physical reality!
- “Wave function of universe” $|\psi_t\rangle = T(t, t_0)|\psi_0\rangle$
 - Everett: $|\psi_t\rangle$ represents physical reality
 - Histories: $|\psi_t\rangle$ is pre-probability: useful for finding Born probabilities; inadequate for others

Histories

- Samples space \mathcal{S} for classical stochastic process
 - Sequence of events
 - Example: coin tossed three times:
 HTH, HTT, HHT, \dots are all different
 - *One and only one* sequence in given experiment
- Other stochastic processes:
 - Random walk
 - Brownian motion
 - Waterloo weather

- Quantum sequence of events for Hilbert space \mathcal{H}

$$Y = F_0 \odot F_1 \odot \dots \odot F_f$$

- F_j projector on ray/subspace of \mathcal{H}
 - “ F_0 at t_0 , F_1 at t_1 , ...”
 - Different F_j *not* (necessarily) related by Schr Eqn
 - Call such a sequence a *history*
- Technical comments:
 - \odot is a modification of \otimes
 - History Y an element of $\mathcal{H} \odot \mathcal{H} \odot \dots \odot \mathcal{H}$.

Families of Histories

- Quantum sample space $\mathcal{S} = \{Y^\alpha\}$ consists of histories
- Simplest interesting situation:
 - Single initial state $[\psi_0] = |\psi_0\rangle\langle\psi_0|$ at t_0
 - $t_j > t_0$: $I = \sum_{\alpha_j} P_j^{\alpha_j}$
- Histories indexed by $\alpha = (\alpha_1, \alpha_2 \dots)$
 $Y^\alpha = [\psi_0] \odot P_1^{\alpha_1} \odot P_2^{\alpha_2} \odot \dots$
- *One and only one* history from $\mathcal{S} = \{Y^\alpha\}$ actually occurs in quantum system starting in $[\psi_0]$ at t_0 .
 - Projectors $P_j^{\alpha_j}$ not related by Schr eqn
- QM does not say *which* history occurs
 - QM can assign probabilities

Probabilities for Histories

- Use isolated/closed system
 - Open systems more complicated
 - Any apparatus is *part* of quantum system
 - in contrast with textbook approach
- Born rule limited to 2-time histories, $t_0 < t_1$
 - New rule needed for 3 or more times
 - Quantum probabilities (usually) not Markovian
- Histories approach *only* assigns probabilities to *consistent* families
 - For consistent families these probabilities
 - make physical sense
 - agree with Born for two times
- Consistency conditions: Chs. 10, 11 of *Consistent Quantum Theory*

Consistency Conditions

- Simplest case: pure initial state $|\psi_0\rangle$, chain kets; see *Consistent Quantum Theory*, Sec. 11.6
- Recursively define

$$\begin{aligned} |\alpha_1\rangle &= P^{\alpha_1}T(t_1, t_0)|\psi_0\rangle, \\ |\alpha_1, \alpha_2\rangle &= P^{\alpha_2}T(t_2, t_1)|\alpha_1\rangle, \\ |\alpha_1, \alpha_2, \alpha_3\rangle &= P^{\alpha_3}T(t_3, t_2)|\alpha_1, \alpha_2\rangle \end{aligned}$$

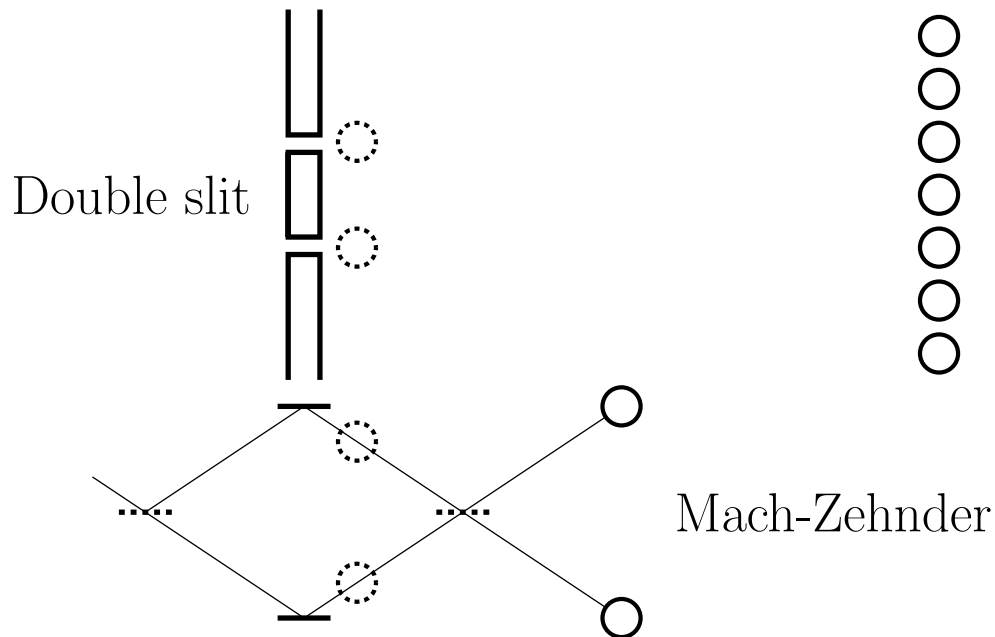
and so forth

- Require orthogonality at each stage:

$$\begin{aligned} \alpha_1 \neq \alpha'_1 &\Rightarrow \langle \alpha_1 | \alpha'_1 \rangle = 0, \\ \alpha_1 \neq \alpha'_1 \text{ OR } \alpha_2 \neq \alpha'_2 &\Rightarrow \langle \alpha_1, \alpha_2 | \alpha'_1, \alpha'_2 \rangle = 0, \\ &\text{Etc.} \end{aligned}$$

- $\text{Pr}(Y^{(\alpha_1, \alpha_2, \dots)}) = \langle \alpha_1, \dots, \alpha_f | \alpha_1, \dots, \alpha_f \rangle$.
- $\{P_1^{\alpha_1}\}$ orthogonal *implies*: $\alpha_1 \neq \alpha'_1 \Rightarrow \langle \alpha_1 | \alpha'_1 \rangle = 0$
 - Two-time $t_0 < t_1$ histories automatically consistent
 - Born rule always works
- Consistency not trivial for 3 or more times.

Double Slit + Mach-Zehnder



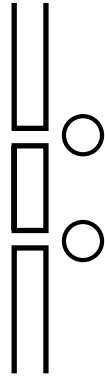
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 - Detectors behind slits \leftrightarrow inside interferometer
 - In interference zone \leftrightarrow following 2d beam splitter
- For precise description, use Mach-Zehnder
 - Basic idea applies to double slit

No Detectors



- Family “Super” (superposition)
 - Initial wave $|\psi_0\rangle$ arrives at slits
 - Ignore reflection (Mach-Zehnder better for this)
 - Passes through slits in *coherent superposition*
- Family “Which” (which slit?)
 - Same $|\psi_0\rangle$
 - Particle passes through definite slit
- Either family gives *valid quantum description*
 - Physicist can choose either. Liberté !
 - Spin 1/2 analogy: Use *either* S_z *or* S_x basis
- Both families equally “fundamental” QM: Egalité !
- Cannot combine Super with Which: Incompatibilité !

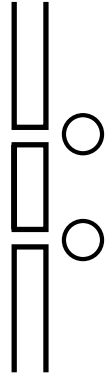
Detectors Behind Slits I



- Detectors are quantum objects!
 - Hilbert space includes detectors
 - Histories include projectors on detector states
 - “Pointer basis”: Macroscopically distinct detector states for decomposition of identity

- Histories: $[\Psi_0] \odot \text{Particle} \odot \text{Detector}$
 - Initial $|\Psi_0\rangle = |\psi\rangle \otimes |\text{Detectors Ready}\rangle$
 - Particle: Super(position) OR Which (slit) basis
 - Detector: Point(er) basis OR Scat (Schr. cat) basis

Detectors Behind Slits II



Family:	Particle:	Detector:	Consistent?
1. Unitary	Super	Scat	Yes
2. Textbook	Super	Point	Yes
3. Exptlist	Which	Point	Yes
4. Nonsense	Which	Scat	No

- Families 1, 2, 3 equally good quantum descriptions!
 1. Individual detectors cannot be discussed.
Misleading concept of “dead and live” cat
 2. “Which slit?” is meaningless question for this family
 3. Experimentalist description: Particle came through slit preceding triggered detector
- Family 4 unacceptable: violates consistency conditions

Detectors in Interference Region



Family:	Particle:	Detector:	Consistent?
1. Unitary	Super	Scat	Yes
2. Textbook	Super	Point	Yes
3. DefinSlit	Which	Scat*	Yes
4. Nonsense	Which	Point	No

- Families 1, 2, 3 equally good quantum descriptions!
 1. Individual detectors cannot be discussed
 2. “Which slit?” is meaningless question
 3. Scat* differs from Scat, but equally odd
- Family 4 unacceptable: violates consistency conditions

Delayed Choice



- Detectors behind slits removed at last moment
 - Does the future influence the past?
- Consistent families in the two cases:

Family:	Particle:	Near Detectors:	Far Detectors:
1. Unitary	Super	Scat	
2. Textbook	Super	Point	
3. Exptlist	Which	Point	
4. Unitary	Super		Scat
5. Textbook	Super		Point
6. DefinSlit	Which		Scat*

- Future influences past = misunderstanding
 - Many *equally valid* quantum descriptions
 - Choosing one does not influence reality; it determines which questions have answers

Einstein Podolsky Rosen

- Phys. Rev. 1935 “Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?”
 - Their answer: No
- Bohm 1952. Particles a and b far apart in spin singlet
 - $|\psi_0\rangle = (|z_a^+ z_b^-\rangle - |z_a^- z_b^+\rangle) / \sqrt{2}$
 - A measures S_{az} , gets result $+1/2$ or $-1/2$
 Conclusion $S_{bz} = -S_{az}$
 - A could measure S_{ax} , get result $+1/2$ or $-1/2$
 Conclusion $S_{bx} = -S_{ax}$
- EPR objection, stated in Bohm language
 - Measurement of a cannot affect b , so
 - S_{bz} values same *before and after* A measurement
 - A could just as well measure S_{ax}
 - Both S_{bz} , S_{bx} have definite values regardless of what is measured
 - QM is incomplete: Hilbert space is too small!

Nonlocality

- Claim: EPR \Rightarrow QM nonlocal!
 - A 's measurement of particle a has an Instantaneous Nonlocal Superluminal (INS) influence on particle b
 - Idea supposedly supported by Bell inequalities
- Laborious analysis proves that
 - INS influences carry no information!
- Histories response:
 - INS influences carry no information — because they do not exist!
 - Bell ineq. violations \Rightarrow hidden variables don't work
 - Hilbert space QM, properly understood, is *local*

Classical Analogy

- Two colored slips of paper: R(ed), G(reen)
- Pete in Pittsburgh
 - Seals them in opaque envelopes
 - Shuffles envelopes
 - Addresses one to Alice in Atlanta
 - The other to Bob and Boston
- Alice opens her envelope, sees G
 - Conclusion: Bob's envelope contains R
- Does this indicate INS influence of Alice's action on Bob's envelope?
 - Perhaps there is some simpler explanation

EPR Correlations

- Histories with initial $|\psi_0\rangle = (|z_a^+ z_b^-\rangle - |z_a^- z_b^+\rangle) / \sqrt{2}$
 - No measurements (until later)
- Unitary family $[\psi_0] \odot [\psi_0] \odot [\psi_0] \odot \dots$
 - Consistent, probability 1
 - Incompatible with individual properties of a, b
 - Must consider $|\psi\rangle$ pre-probability in order to use reduced density operators ρ_a, ρ_b
- Family using S_z bases: $[\psi_0] \odot \begin{cases} z_a^+ z_b^- \\ z_a^- z_b^+ \end{cases}$
 - Perfect correlations with no measurements
 - Like R, G slips of paper inside envelopes
 - Good measurements will show what is there
- Family using S_x bases: $[\psi_0] \odot \begin{cases} x_a^+ x_b^- \\ x_a^- x_b^+ \end{cases}$
 - Incompatible with previous family
- No magical INS influences thus far!
 - Will measurements bring them to light?

EPR Measurement Correlations

- Measuring apparatus Z_a for S_{az}
 - Initial state $|\Psi_0\rangle = |\psi_0\rangle \otimes |Z_a^0\rangle$
 - Pointer basis projectors Z_a^+, Z_a^-
- Unitary history \rightarrow apparatus Schrödinger cat state
 - Good QM, but does not address our questions
- Family 1. $[\Psi_0] \odot \begin{cases} z_a^+ z_b^- \odot Z_a^+ \\ z_a^- z_b^+ \odot Z_a^- \end{cases}$
 - Apparatus Z_a^\pm correlated with prior states of both particles
 - Good measurements show what is there
- Extensions of this family show that:
 - Z_a^+ outcome implies $S_{bz} = -1/2$ for particle b *before, during* and *after* measurement
 - No sign of INS influence!

EPR Delayed Choice

- Measure S_{ax} instead of S_{az}
 - Use $|\Psi_0\rangle = |\psi_0\rangle \otimes |X_a^0\rangle$
 - Or use quantum coin to replace Z_a with X_a
 - Possibly at the last moment

- Family 2. $[\Psi_0] \odot \begin{cases} x_a^+ x_b^- \odot X_a^+ \odot x_b^- \\ x_a^- x_b^+ \odot X_a^- \odot x_b^+ \end{cases}$
 - Outcome X_a^+ correlated with x_b^- — both before and after measurement

- Family 3. $[\Psi_0] \odot \begin{cases} z_b^+ \odot \{X_a^+, X_a^-\} \odot z_b^+ \\ z_b^- \odot \{X_a^+, X_a^-\} \odot z_b^- \end{cases}$
 - Can discuss S_{bz} when S_{ax} measured, why not?
 - S_{bz} for particle b *exactly the same* before and after measurement on particle a
 - Demonstration of *absence* of INS influences!

Is QM Complete?

- Quantum description of physical reality *available in 1935* was *incomplete* because it:
 - Lacked consistent probabilities
 - Limited set of stochastic descriptions
 - Misleading reliance on “measurement”
 - Lacked good description of measurement apparatus
 - Wavefunction “collapse” not well formulated
- Einstein, Podolsky, Rosen correct in raising objections
- Quantum description *available in 2005* has
 - Consistent system of probabilities for microsystems
 - Broad class of stochastic descriptions
 - Formulation does not rely on measurement
 - Same principles for measurements, other processes
 - Conditional probabilities replace “collapse”
- Is it now complete?

Discussion Topics

- Liberty in choosing alternative descriptions
 - Analogy of ordinary historian
 - Approach does not lead to contradictions
- Measurements
 - Reveal pre-existing properties
 - if included in framework
- Von Neumann measurements
 - Very special type
 - Usual interpretation is not wrong
 - but it is misleading
- Approximate consistency: Dowker and Kent
- Consistent histories vs. Everett
 - $|\psi_t\rangle$: pre-probability, not reality
- Consistent histories vs. Bohm
 - Particle triggers detector by not passing through it