What Is Quantum Information?

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- References (work by R. B. Griffiths)
  - “Nature and location of quantum information.”
  - “Channel kets, entangled states, and the location of quantum information,”
  - *Consistent Quantum Theory* (Cambridge 2002)
    http://quantum.phys.cmu.edu/
Introduction

• What is quantum information? Precede by:
  ○ What is information?
  ○ What is classical information theory?

• What is information? Example, newspaper
  ○ Symbolic representation of some situation
  ○ Symbols in newspaper *correlated* with situation
  ○ Information is *about* that situation
Classical Information Theory

- Shannon:
  - “Mathematical Theory of Communication” (1948)
  - One of major scientific developments of 20th century
  - Proposed *quantitative* measure of information

- Information entropy

\[
H(X) = - \sum_i p_i \log p_i
\]

- Logarithmic measure of missing information
- *Probabilistic* model: \( \{p_i\} \) are probabilities
- Applies to classical (macroscopic) signals

- Coding theorem: Bound on rate of transmission of information through noisy channel
Quantum Information Theory (QIT)

- Goal of QIT: “Quantize Shannon”
  - Extend Shannon’s ideas to domain where quantum effects are important
  - Find quantum counterpart of $H(X)$

- We live in a quantum world, so
  - QIT should be the *fundamental* info theory
  - Classical theory should emerge from QIT
  - Analogy: relativity theory $\rightarrow$ Newton for $v \ll c$
QIT: Current Status

- Enormous number of published papers
  - Does activity = understanding?

- Some topics of current interest:
  - Entanglement
  - Quantum channels
  - Error correction
  - Quantum computation
  - Decoherence

- Unifying principles have yet to emerge
  - At least, they are not yet widely recognized
QIT: Proposals

Published answers to question:
What is Quantum Information?

- Bennett and Shor (1998)
  ○ Qm ↔ Cl info is like complex ↔ real numbers
    – Interesting analogy, but what are the details?

- Deutsch and Hayden (2000)
  ○ Technical idea using Heisenberg representation
    – Causality, not information in Shannon sense

- Brukner and Zeilinger (2001)
  ○ Shannon ideas don’t work in quantum domain
    – But they do, if quantum probabilities correctly defined

- Caves, Fuchs, Schack (2002 and later)
  ○ Quantum wavefunction represents our knowledge
    – Our knowledge of what?
QIT: Problem

- Quantizing Shannon faces a fundamental problem
  - Shannon theory based on *probabilities*
  - What *quantum* probabilities to put in formulas?

- Textbook (Copenhagen) quantum mechanics:
  - Probabilities $\leftrightarrow$ *measurement outcomes*
  - Measurement outcomes are macroscopic (classical)
  - Measurements *do not* reveal quantum properties
  - So provide no basis for *quantum* information

- Solution: Modernize the textbooks!
  - Measurements *do* reveal quantum properties
  - *Quantum* probabilities possible *if* one uses a *consistent* formulation of quantum theory!
Consistent Quantum Information

- Histories approach to quantum probabilities
  - Developed by Gell-Mann, Griffiths, Hartle, Omnès
  - Precise mathematical, logical rules
    - Apply to quantum systems of any size
    - Consistent results; always know what you’re doing

- Naive quantum probability assignments
  - Result in paradoxes, mysteries, confusion

- Histories approach employs:
  - Many frameworks for quantum probabilities
  - Consistent probabilities in each framework
  - Cannot combine probabilities from incompatible frameworks

- Different incompatible frameworks ↔ different types / kinds / species of quantum information
  - Different species cannot be combined!

- Will use “multiple species” approach to discuss
  - “One bit” and ordinary (two bit) teleportation
  - Decoherence
Spin 1/2 Example

- Measure $S_x$ using Stern-Gerlach
  - Result is $S_x = +1/2$ or $S_x = -1/2$ (units of $\hbar$)
  - Measurement *outcome* (pointer position) provides information about $S_x$ before measurement took place.
  - Call this $X$ *information* about the particle

- Measure $S_z$ using Stern-Gerlach
  - Result is $S_z = +1/2$ or $S_z = -1/2$
  - Call this $Z$ *information* about the particle

- $X$ info and $Z$ info are *incompatible*, different species, they cannot be combined.

- “$S_x = +1/2$ AND $S_z = +1/2$” is *meaningless*
  - Corresponds to nothing in Hilbert space
  - So quantum mechanics assigns it no meaning

- “$S_x = +1/2$ OR $S_z = +1/2$” is also meaningless

- “$S_x = +1/2$ AND $S_x = -1/2$” is meaningful, FALSE
  - Just one kind or species of information involved

- “$S_x = +1/2$ OR $S_x = -1/2$” meaningful and TRUE
Classical Information

- Macroscopic object with angular momentum $L$
  - One can measure $L_x$ or $L_z$
  - Different pieces of information, same species
  - Can be combined in a meaningful way
  - “$L_x = 5$ Js AND $L_z = 7$ Js” makes sense

- We live in a quantum world!
  - Need only one quantum info species for macro world
  - By convention, this species is “classical information”
  - There is no classical information that is not some sort of quantum information.
Quantum Channel

- One-qubit (spin-half particle) quantum channel
  \[
  |\psi\rangle \xrightarrow{\text{Pipe}} |\psi\rangle
  \]

- Perfect channel: \( |\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle \)
  - \( Z \) info: \( S_z = +\frac{1}{2}, -\frac{1}{2} \leftrightarrow |\psi_{\text{in}}\rangle = |0\rangle, |1\rangle \)
  - \( X \) info: \( S_x = +\frac{1}{2}, -\frac{1}{2} \leftrightarrow |\psi_{\text{in}}\rangle = |+\rangle, |-\rangle \)
  - *Only one* species goes through at one time
  - *Any* species is correctly transmitted

- Long distance transmission problem

  \[
  |\psi\rangle \xrightarrow{\text{Measure } S_z} \xrightarrow{\text{Cl channel}} |\psi\rangle?
  \]

  - Can send \( Z \) information, but not \( X \) information
  - Alternative measurement: send \( X \) info, not \( Z \)

- Cannot make Qm channel using Cl channel
  - Cl channel transmits *only one* species of Qm info
  - Qm channel transmits *all* species
**Teleportation**

- **Teleportation**: Bennett et al. (1993)

  \[ |\psi\rangle \quad \text{Measurement} \]

  - **Requirements:**
    - Shared entangled state already exists
    - Correlated measurement of two qubits
    - Two bits to send measurement outcome to \( B \)
    - Two unitary corrections by \( B \)

- **Why two classical bits? Why not one? or three?**

- **One-bit Teleportation**: Zhou et al. (2000)

  \[ |\psi\rangle \quad \text{One classical bit} \]

  - No entangled, state, only one classical bit, but
  - Requires nonlocal CNOT gate between \( A \) and \( B \)
One-Bit Teleportation: $Z$ Information

- Circuit: Original and Quantized

\[ \begin{align*}
|\psi\rangle & \quad A & H & \quad |\psi\rangle \\
|0\rangle & \quad B & Z & \quad |\psi\rangle \quad \quad \quad \quad \quad |0\rangle & \quad B & Z & \quad |\psi\rangle \\
\end{align*} \]

- $Z$ information ($S_z$) about initial state of $A$ qubit:
  - $Z$ info = difference between initial $|0\rangle$ and $|1\rangle$
  - Is \textit{copied} to $B$ by the CNOT gate
  - Is unaffected by final $Z$ gate:
    \[
    Z|0\rangle = |0\rangle, \quad Z|1\rangle = -|1\rangle
    \]
    (phase $-1$ is unimportant for $Z$ information)

- Conclusion: $Z$ info arrives unaltered at output
  - Even if classical bit (or quantum counterpart) omitted
One-Bit Teleportation: $X$ Information

- $X$ information ($S_x$) about initial state of $A$ qubit:
  - $X$ info = difference between initial $|+\rangle$ and $|--\rangle$

- CNOT puts it in *correlation* between $A$ and $B$ qubits:
  
  \[
  |+\rangle \rightarrow \frac{1}{2}(|++\rangle + |--\rangle), \quad S_{Ax} = S_{Bx}
  \]
  
  \[
  |--\rangle \rightarrow \frac{1}{2}(|+-\rangle + |-+\rangle), \quad S_{Ax} = -S_{Bx}
  \]
  - $X$ info *not* present in individual $A$, $B$ qubits

- Hadamard $H$ gate converts $X$ to $Z$:
  - Case $|+\rangle \rightarrow |0\rangle$, no correction needed
  - Case $|--\rangle \rightarrow |1\rangle$, final $Z$ gate changes $|+\rangle \leftrightarrow |--\rangle$

- Conclusion: $X$ info arrives unaltered at output
  - Classical bit (or quantum counterpart) is essential
  - $X$ info not in classical bit *by itself*
Information In Correlations

• Information in correlations is a classical, not a special quantum concept

• Illustration: C sends message by
  ○ Mailing colored slips of paper to A and B
  ○ Colors are red (R) or green (G)
  ○ Message 0: Same color (RR or GG) to A, B
  ○ Message 1: Different colors (RG or GR) to A, B

• Neither A nor B *individually* can read the message
  ○ Information of 0 vs. 1 is in *correlation* of colors
“Presence” Theorem

• One-bit teleportation works for $Z$ info and $X$ info
  ○ What about $Y$ info? Other species?
  ○ We don’t need to check them all because of the:

• Presence Theorem (qubits): If *any two incompatible* species of information are correctly transmitted from input to output, the same is true of all species.
  ○ Theorem applies to noise-free transmission
  ○ Refers to two incompatible species, so this is a quantum information theorem, no classical counterpart

• Generalization to $d$-dimensions: If *two* “sufficiently incompatible” species are correctly transmitted, all species are correctly transmitted (channel is perfect).
  ○ Example: two orthonormal bases $\{|a_j\rangle\}$ and $\{|\bar{a}_k\rangle\}$, with $\langle a_j | \bar{a}_k \rangle \neq 0$ for all $j, k$, are sufficiently incompatible
Regular (2 Bit) Teleportation

- Circuit:

\[
\begin{array}{c}
|\psi\rangle \rightarrow c \\
\xrightarrow{H} \\
\xrightarrow{Z \text{ info}} \\
\xrightarrow{X \text{ info}} \\
\xrightarrow{X} \\
\xrightarrow{Z} \\
\xrightarrow{|\psi\rangle} \\
\end{array}
\]

\[
\frac{|00\rangle + |11\rangle}{\sqrt{2}}
\]

- One “classical” bit carries $Z$, the other $X$ information
  - Use only “$X$” bit: will transmit $X$ info
  - Use only “$Z$” bit: will transmit $Z$ info
  - Quantization of circuit left as exercise

- Each species of information is in a *correlation* between the “classical” bit and the $b$ qubit
  - Measuring classical bit tells one nothing
  - Measuring the $b$ qubit tells one nothing

- Teleportation needs 2 classical bits because
  - There is *more than one* species of quantum info
  - If *two* species correctly transmitted, others follow
    - So do not need three (or more) bits

- $d$-dimensional teleportation (qudit): same argument
  - Two incompatible species $\Rightarrow 2 \log_2 d$ classical bits
Decoherence: Introduction

- Decoherence results when a quantum system interacts with its (quantum) environment
- Old perspective: Off-diagonal elements of density matrix go to zero
- New perspective (Zurek): *Information* flows from system to environment
- What can we learn using incompatible species of info?
Example: Interferometer

- No decoherence, particle initially in $|p+\rangle$
  
  $|p+\rangle := (|p_0\rangle + |p_1\rangle)/\sqrt{2} \rightarrow |q_0\rangle$
  
  $|p-\rangle := (|p_0\rangle - |p_1\rangle)/\sqrt{2} \rightarrow |q_1\rangle$

  - Particle emerges in $q_0$, not $q_1$, because of *coherence*

- Decoherence:
  - Which path, $|p_0\rangle$ vs $|p_1\rangle$, info $\rightarrow$ environment
  - Particle emerges randomly in $q_0$ or $q_1$

- Interpretation using different incompatible species:
  - $Z$ (which path) info: $|p_0\rangle$ vs $|p_1\rangle$
  - $X$ (which phase) info: $|p+\rangle$ vs $|p-\rangle$
    - Decoherence means $X$ (which phase) information has vanished when particle exits interferometer

- $Z$ info in environment $\Rightarrow X$ info absent at output
  - Consequence of Absence Theorem
“Absence” Theorem

• Theorem. Three systems $A$, $B$, $C$. If $Z$ info about $A$ is *present* in $B$, then $X$ info about $A$ is *absent* from $C$.

  ○ Two incompatible species; this is a *quantum* information theorem

  ○ Present = perfectly present, Absent = perfectly absent

• Application to decoherence:

  • If which path ($Z$) info about particle ($A$) entering interferometer is in the environment ($B$), coherent ($X$) info not present in particle ($C$) exiting interferometer, so there is *no interference*

  ○ Particle at earlier ($A$) and later ($C$) times can be thought of as two systems when applying the theorem
General “Absence” Theorem

- Three systems: $A, B, C$. Dimension $d$ of $A$ arbitrary. $\mathcal{Z} = \{|a_j\rangle\}$, $\mathcal{X} = \{|\tilde{a}_k\rangle\}$ mutually unbiased bases of $A$.
- Theorem. If $\mathcal{Z}$ info about $A$ is *present* in $B$, then $\mathcal{X}$ info about $A$ is *absent* from $C$.

- Present = perfectly present, Absent = perfectly absent
- There may be better ways of wording the theorem
Decoherence: Conclusion

- If a particular species (“pointer basis”) of information about the (earlier) state of a quantum system is available at some place in the environment, then other maximally-incompatible species of information about the same system will not be present at other places in the environment, or in the system itself.
  - (Pace Zurek) It does not matter how many different places in the environment the information is located.
  - It must be “clearly” present in (at least) one place.
  - Generalization of Absence Theorem to partial presence or absence would be worthwhile.
Summary

- By distinguishing different incompatible species we can:
  - Trace information flow in teleportation
    - See why 2 classical bits are needed
    - Or 1 classical bit for 1-bit teleportation
  - Understand decoherence as a process in which spreading one species of information excludes others

- Open issues:
  - Extend “Presence”, “Absence” theorems to *partial* presence/absence
  - Can information species be used to clarify asymptotic properties (channel capacities of various sorts)?